Problem of the Week

Solution to Number Three

PROBLEM: Evaluate:

$$(1 - \cot 23)(1 - \cot 22).$$

The angle measures are in degrees.

SOLUTION: Note that

$$1 = \cot(45) = \cot(23 + 22) = \frac{(\cot 23)(\cot 22) - 1}{\cot 23 + \cot 22}.$$

This implies that

$$\cot 23 + \cot 22 = \cot 23 \cot 22 - 1.$$

It follows that

$$[(\cot 23)(\cot 22) - \cot 23 - \cot 22 + 1] - 2 = (1 - \cot 23)(1 - \cot 22) - 2 = 0.$$

Therefore, we have

$$(1 - \cot 23)(1 - \cot 22) = 2.$$