Problem of the Week

Number Six

February 18, 2013

This week's problem comes from an old episode of *Columbo*. So, if you have not heard it before, that just means you need to watch more television.

You are given five bags. Four of them contain five gold coins each, while the remaining bag contains five coins made of counterfeit gold. Sadly, you cannot tell just by looking which bag contains the fakes. Happily, you just happen to know that the counterfeit coins are slightly heavier, by a known amount, than the real coins. Sadly, this amount is not sufficient for you to identify the fakes just by feel. Happily, you have a highly accurate scale at your disposal. Sadly, you are only allowed to use the scale once. Happily, it turns out that one weighing is sufficient to determine which bag contains the fakes!

Your problem is to show me how to determine, conclusively, which bag contains the fakes, while using the scale only once.

This is our sixth problem of the week. Of course, 6 is a fine number indeed, since it is the smallest perfect number. It is also the only triangular number whose square is triangular as well. Cool, no? (A number is perfect if it is equal to the sum of its proper divisors. $6 = 1 + 2 + 3$, you see. A number is triangular if for some value of n it is equal to the sum of the first n consecutive integers. We have just seen that 6 is traingular, and since $36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$, we see that it is triangular as well.

Solutions are due to Jason Rosenhouse by 5:00 on Friday, February 1. One weekly winner will receive a five-dollar gift card from Starbucks. Winners will be drawn randomly from among the correct answers. The solution will be posted at the POTW website: