I have a remarkable plant. It grows very rapidly, you see. At the end of the first day it had increased its height by a factor of $\frac{1}{2}$. At the end of the second day it had increased its height by a factor of $\frac{1}{3}$ beyond where it was at the end of the first day. At the end of the third day it had increased its height by a factor of $\frac{1}{4}$ beyond where it was at the end of the second day. This pattern continues, so that by the end of the n-th day, the plant has increased its height by a factor of $\frac{1}{n+1}$ beyond where it was at the end of the day before. How many days did it take to grow to one hundred times its original height?

SOLUTION: For simplicity, let's assume that the plant was initially one foot tall (though the exact number is irrelevant.) After the first day it has increased its height by a factor of $\frac{1}{2}$, meaning that it has grown $\frac{1}{2}$ foot and is now $\frac{3}{2}$ feet tall. At the end of the next day it increases its height by a factor of $\frac{1}{3}$, meaning that it is now $\left(\frac{3}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{2}$. So, the plant has again grown by $\frac{1}{2}$ foot.

Continuing this patttern, we find that after n days the plant is $\frac{n+1}{2}$ feet tall, and then increases its height by a factor of $\frac{1}{n+1}$. In other words, at the end of each day the plant has added half a foot to its height. It will be one hundred times its original height when it had grown by 99 feet. At half a foot per day, this will require 198 days, and that is the answer.