Problem of the Week Solution Three

Which of the following statements is/are true?

- 1. Exactly one of these statements is false.
- 2. Exactly two of these statements are false.
- 3. Exactly three of these statements are false.
- 4. Exactly four of these statements are false.
- 5. Exactly five of these statements are false.
- 6. Exactly six of these statements are false.
- 7. Exactly seven of these statements are false.
- 8. Exactly eight of these statements are false.
- 9. Exactly nine of these statements are false.
- 10. Exactly ten of these statements are false.

SOLUTION: Only statement nine is true. To see this, notice that it is impossible for statement ten to be true. If it is true, then it must be the case that all ten statements are false which is an immediate contradiction. Therefore, at least one of these statements is true. But since the statements all directly contradict one another, it is impossible for more than one of these statements to be true. It follows that there are exactly nine false statements and one true statement, which is precisely what statement nine says.