
Problem of the Week

Solution to Problem Six

You are given five bags. Four of them contain five gold coins each, while the remaining bag contains five coins made of counterfeit gold. Sadly, you cannot tell just by looking which bag contains the fakes. Happily, you just happen to know that the counterfeit coins are slightly heavier, by a known amount, than the real coins. Sadly, this amount is not sufficient for you to identify the fakes just by feel. Happily, you have a highly accurate scale at your disposal. Sadly, you are only allowed to use the scale once. Happily, it turns out that one weighing is sufficient to determine which bag contains the fakes!

Your problem is to show me how to determine conclusively which bag contains the fakes, while using the scale only once.

SOLUTION: The trick is to remove one coin from the first bag, two coins from the second bag, three from the third, four from the fourth, and all five from the fifth. This makes a total of fifteen coins. Let x be the weight of a real coin, and let y represent the amount by which the weight of a fake coin exceeds the weight of a real coin. Then your scale will give you a reading of $15x + ky$ where k is an integer between one and five inclusive. From this you can determine the number of fake coins on the scale, which in turn will tell you which bag contains the fakes.