Mr. Smith keeps all his socks in one drawer. Each morning, he pulls out two of them at random. Wearing unmatched socks does not bother him, you see, so he just takes his chances. At night those two socks are returned to the drawer, so that he repeats precisely the same procedure each day. (Apparently he also does not care about wearing dirty socks!) If he has four pairs of socks – one black, one white, one red and one blue – how many days per week, on average, will he be found wearing a matching pair?

SOLUTION: Suppose that Mr. Smith draws his first sock at random. Then there will be seven socks remaining in the drawer, only one of which matches the sock he drew first. That gives him a probabiility of $\frac{1}{7}$ of getting a matching pair. It follows that, on average, he will have a matching pair on one day out of each week.

An alternative approach is to ask about the probability that both socks are a particular color, say red. The probability that the first sock will be red is $\frac{2}{8}$. Having selected that first sock, the probability that the second sock will be red is now $\frac{1}{7}$. Thus, the probability that both socks will be red is found by multiplying these together. And since there are four colors, the probability that both socks match will be given by

$$4\left(\frac{2}{8}\right)\left(\frac{1}{7}\right) = \frac{1}{7}.$$