Problem of the Week

Solution Nine

Suppose a steel beam, one mile long, is fastened securely to the ground at each end. As the day heats up, the metal expands. Let us assume that at the hottest part of the day, the metal is actually one mile and one foot long. Let us further assume that the beam is fastened in such a way that it can only buckle upward, and not side to side. Your problem is to estimate how high the beam will be above the ground. Would you be able to slip a playing card under it? A pencil? Would you be able to crawl under it? Walk under it? Could an elephant (roughly 13 ft tall) walk under it? How about a person standing on the back of an elephant? That sort of thing. Be sure to justify your answer!

SOLUTION: Since we are only interested in approximating the height in some reasonable way, we shall picture the heat-expanded beam as an isosceles triangle. The base of the triangle is the original beam, one mile long, while the expanded beam is represented by the two sides of the triangle, each of equal length. The altitude of the triangle to the base from the vertex in the air is the length we seek.

A mile is 5280 feet, but we will just use 5000 feet as an approximation. Then we apply the Pythagorean theorem to the height, half the base, and one side of the isosceles triangle. Half the base will be 2500 feet, while one of the sides will have length $2500 + \frac{1}{2}$ feet. Then the height, h, must be

$$\begin{aligned} h &= \sqrt{\left(2500 + \frac{1}{2}\right)^2 - (2500)^2} \\ &= \sqrt{2500 + \frac{1}{4}} \approx 50. \end{aligned}$$

So, at its highest point the beam willbe roughly fifty feet above the ground. Goodness!