
Problem of the Week

Number Ten

April 21, 2014

Okay folks. This is our final problem. Time to bring it all home. I've got a real humdinger for you this week. Don't even read it, unless you are equipped with the special safety glasses. Here goes.

Recall that we have three kinds of people. There are knights, who only make true statements and knaves, who only make false statements. There are also people in the transitional phase. They are called "Neutrals" and only make statements with the truth value N. Recall that if Joe, say, is in the transitional phase, and you make the statement, "Joe is a knight," or "Joe is a knave," then your statement receives the truth value N. Such statements, and statements built from them by the standard logical connectives, are the only kinds of statements that can receive the truth value N.

The truth tables for the standard connectives are as follows:

\wedge	T	N	F	\vee	T	N	F
T	T	N	F	T	T	T	T
N	N	N	F	N	T	N	N
F	F	F	F	F	T	N	F

\rightarrow	T	N	F	\leftrightarrow	T	N	F
T	T	N	F	T	T	N	F
N	T	T	N	N	N	T	N
F	T	T	T	F	F	N	T

The first two tables show how to handle "and" statements and "or" statements. The second two deal with "if-then" statements and "if and only if" statements. In each table, the truth value of the first part of the statement appears down the side, while the truth value of the second part appears across the top. So, for example, our tables tell us that if P is neutral and Q is true, then the statement $P \wedge Q$ is neutral. Let me remind you of the equivalence:

$$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P).$$

Thus, given the tables for \rightarrow and \wedge , we can quickly fill in the table for \leftrightarrow . There are reasons for filling in the tables as we have, but we do not need to go into that here. For our purposes, these are just the rules of the game.

This week shall also see the return of the normals. Recall that normals sometimes make true statements and sometimes make false statements. However, they never make statements with the truth value N. So, in this week's problem, we have knights, knaves, normals and neutrals. In what follows, I shall use the standard abbreviation "iff" for "if and only if." Here's the problem:

You meet eight people. You know that there are two knights, two knaves, two normals and two neutrals in the group. They make the following statements:

Godzilla : King Kong is a neutral or Juggernaut is a neutral.

House : Lear is a knave and MacBeth is a knave.

Inspector Queen : If Godzilla is a knight, then House and MacBeth are normals.

Juggernaut : I am not a neutral.

King Kong : If Juggernaut is not a neutral then neither is House.

Lear : House is a knight and Godzilla is a knave.

MacBeth : Inspector Queen is a knight iff Nero Wolfe is a neutral.

Nero Wolfe : Lear is a neutral iff King Kong is not a neutral.

Determine the types of all eight people.

Good luck with that! Congratulations in advance to anyone who manages to solve it.

*Solutions are due to Jason Rosenhouse by 5:00 on Friday, April 25. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. **Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible.** Problems are available at the bulletin board outside Roop 119, and also at the website:*

<http://educ.jmu.edu/~rosenhjd/POTW/Spring14/homepage.html>