Problem of the Week
Number Two
January 27, 2014

In calculus, a vector is said to be "normal" to a surface at a point if it is perpendicular to the surface at that point. Have you ever wondered about the origin of that term? After all, of the infinitely many directions in which the vector can go, only one is the direction of perpendicularity. Perhaps that means a vector travelling in that direction should be referred to as an "abnormal" vector.

Well, according to Steven Schwartzman's excellent book *The Words of Mathematics*, the word "normal" comes from the Latin word *norma*, which refers to a standard measure of some kind. The word also referred specifically to a carpenter's square, which is a tool for drawing right angles. Hence the connection between the word "normal" and perpendicularity.

Fascinating stuff, but entirely irrelevant to this week's problem. The "normals" we have in mind are a group of people who recently visited the island of knights and knaves. Recall that knights only make true statements while knaves only make false statements. Normals, on the other hand, sometimes make true statements and sometimes make false statements. Just like normal people! For residents of the island, it was customary to say that knights were of "higher rank" than normals, and that normals, in turn, were of "higher rank" than knaves. That's the set-up. Here's the problem:

You meet three people, whose names are Data, Ezri, and Fell. One of them is a knight, one is a knave, and one is a normal. Data says, "Ezri is of higher rank than Fell." Ezri then says, "Fell is of higher rank than Data." If you now ask Fell, "Who has higher rank, Data or Ezri?" how will Fell reply?

Schwartzman's book makes for fascinating reading. Did you know, for example, that the words "parable" and "parabola" actually have the same root? Both come from Greek words that translate roughly as "to thrust up against, or "to put alongside." In the theory of conic sections, parabolas have eccentricity one, which thrusts them right up against the boundary value separating ellipses from hyperbolas. A "parable," meanwhile, is a story that brings clarity to a complex situation by placing it alongside a simpler one.

"Theater" and "theorem" likewise have the same origin. They both come from a Greek word meaning "to look at." A theater is where you go to look at a dramatic production of some kind, while a theorem is something that emerges in a moment of clarity after prolonged "looking at."

Etymology is fun, and so is this week's problem. So get to work! (But not during class, because that's rude.) And please notice what is written on the other side of the page $\implies$

Solutions are due to Jason Rosenhouse by 5:00 on Friday, January 31. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. **Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible.** *Problems are available at the bulletin board outside Roop 119, and also at the website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring14/homepage.html