Problem of the Week
Number Three
February 3, 2014

This week's problem features a conditional statement, which is to say a statement of the form, "If P then Q." Classical logic treats conditionals as *truth-functional*. This means that the truth of a conditional statement depends solely on the truth of the statements P and Q. More specifically, if P is true and Q is false, then the statement, "If P then Q" is deemed to be false. In any other scenario, the statement, "If P then Q" is deemed to be true.

This has some interesting consequences. One is that a statement like, "If $2 + 2 = 5$ then pigs can fly" is deemed to be true. Are you OK with that? There's more. Recall that if a statement of the form, "P and R" is true, then it must be that P and R are both true individually. That means that if the statement, "If P then Q" is true, then it must also be that case that "If (P and R) then Q" is also true. That sounds reasonable enough, until you consider an example like this: Granting that "If it does not rain tonight, then I will play baseball tomorrow," it follows that "If it does not rain tonight and I break my ankle walking down the stairs, then I will play baseball tomorrow." Isn't logic fun?

And how about this one? Keeping in mind that I am in Harrisonburg, Virginia as I write this, what are we to make of the statement, "If I am not in France then I am not in Spain"? Classical logic says that statement is true, since, as it happens, it is true that I am not in France and also true that I am not in Spain. Is this acceptable? Plainly not. It is considerations such as this that has led to the development of so-called "non-classical logics," but that's getting a bit too far afield.

Here is this week's problem:

This time you meet Gandalf, Harry Potter and the Incredible Hulk. Gandalf says, "Harry Potter is a knight." Harry Potter says, "If Gandalf is a knight then so is the Incredible Hulk." Deduce as much you can regarding the knight-hood or knave-hood of the three people.

Chew on that one for a while. Also, please notice what is written on the other side of the page $\Longrightarrow$

Solutions are due to Jason Rosenhouse by 5:00 on Friday, February 7. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. ***Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible.*** *Problems are available at the bulletin board outside Roop 119, and also at the website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring14/homepage.html