Problem of the Week
Number Seven
March 17, 2014

To this point all of our puzzles have involved classical logic, which is based on the idea that there are exactly two truth values: True and False. However, philosophers have long recognized that there are cases for which a mere two truth values is rather confining.

For example, some statements are vague. There are people who are very tall and people who are very short, but there is also a large middle-ground of people who are neither tall nor short. If Joe is of average height and you say, "Joe is tall," it seems arbitrary to declare that statement definitively true or definitively false. Perhaps, then, we need a third truth value to handle cases of vagueness. We could have the truth values T, F and N, say, which stand for true, false and neutral.

You might think this is of purely theoretical interest, but no! You see, a team of biologists living on the island of knights and knaves recently discovered–and I'm sure you can imagine their shock–that knighthood and knavehood were not permanent conditions. Instead, people cycled between the two states. They would be knights for a while, a time during which they made only true statements, and then they would gradually transition to knavehood. These cycles occurred naturally and unpredictably throughout their lives.

Thus, we now had three kinds of people. There were knights, who only made true statements and knaves, who only made false statements. But there were also people in the transitional phase. They were called "Neutrals" and only made statements with the truth value N. If Joe is in the transitional phase, then the statements, "Joe is a knight," or "Joe is a knave," are assigned the truth value N. Of course, true and false continue to mean what they have always meant. The idea is that people in the transitional phase are partly knight and partly knave, suggesting that our statements are vague. Thus, they should not be declared definitively true or definitively false.

As you might suspect, this can lead to some puzzling situations:

While out for a walk, you meet Zod, Adler and Banacek. Zod says, "Adler is a knight." Adler says, "Banacek is a knave." Banacek says, "Zod is a neutral." What can you deduce about the types of the three people?

To help get you started, notice that the statement, "Zod is a neutral," must be either true or false. It is not the sort of statement that can have the truth value N, since there is no ambiguity regarding one's status as a neutral. You cannot be partly neutral and partly non-neutral. Rather, you either are a neutral or you are not.

Good luck! Also, please notice what is written on the other side of the page $\Longrightarrow$

Solutions are due to Jason Rosenhouse by 5:00 on Friday, March 21. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. ***Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible.*** *Problems are available at the bulletin board outside Roop 119, and also at the website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring14/homepage.html