
Problem of the Week

Number Eight

March 24, 2014

This week we continue our exploration of non-classical knights and knaves. Recall how we set things up last week. We imagined that on the island of knights and knaves, everyone cycled, repeatedly and unpredictably, between knight-hood and knave-hood. If Joe is in the transitional phase, then the statements, “Joe is a knight,” and “Joe is a knave,” cannot be said to be definitively true or definitively false. We therefore assign him a new, third truth value, which we denote by “N.” We also noted that people in the transitional phase only make statements with the truth value N. (To this point, the only such statements that are possible are attributions of knight-hood and knave-hood to people in the transitional phase.)

Thus, we now have three kinds of people. There are knights, who only make true statements and knaves, who only make false statements. There are also people in the transitional phase. They are called “Neutrals” and only make statements with the truth value N.

The island’s philosophers now needed to decide how to handle compound statements. In particular, we must relate the truth value of an if-then statement to the truth values of the individual parts. In classical logic such a statement is deemed to be false only when the first part is true and the second part is false. The statement is deemed to be true in all other cases. For three-valued logic, the agreed upon truth table is this:

\rightarrow	T	N	F
T	T	N	F
N	T	T	N
F	T	T	T

This table should be read as follows: If your statement has the form, “If P then Q,” then the truth of P is represented down the side and the truth of Q is represented across the top. So if you only look at the classical truth values (T and F), then this table is identical to the table for classical logic. The truth value “N” should be interpreted as “possibly true and possibly false.” So a statement of the form, “If True then Neutral,” should be considered neutral, while a statement of the form, “If False then Neutral,” should be considered true. We want our three-valued table to preserve as much of our classical intuition as possible. The weirdest entry is the middle one. A statement of the form, “If neutral then neutral” is deemed to be true. This is because a statement such as, “If

Joe is a knight then Joe is a knight,” really ought to be true, even if the statement “Joe is a knight” is itself neutral.

So here’s the problem:

You come to a fork in the road, and want to know which path will take you to the city. Chewbacca is standing nearby so you ask him which path to take. He replies, “If I am a knight then you should take the left fork. But if I am knave then you should take the right fork.” Which path should you take?

Good luck with that! Everything may seem a little weird at first, but if you give it a try you might find that this is not quite as complex as it seems. Feel free to stop by my office or to send me an e-mail if you have questions.

*Solutions are due to Jason Rosenhouse by 5:00 on Friday, March 28. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. **Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible.** Problems are available at the bulletin board outside Roop 119, and also at the website:*

<http://educ.jmu.edu/~rosenhjd/POTW/Spring14/homepage.html>