## Problem of the Week Number Nine April 7, 2014

This week we continue our exploration of non-classical knights and knaves. Recall how we have set things up. We imagined that on the island of knights and knaves, everyone cycled, repeatedly and unpredictably, between knight-hood and knave-hood. If Joe is in the transitional phase, then the statements, "Joe is a knight," and "Joe's is a knave," cannot be said to be definitively true or definitively false. We therefore assign them a new, third truth value, which we denote by "N." We also noted that people in the transitional phase only make statements with the truth value N. (To this point, the only such statements that are possible are attributions of knight-hood and knave-hood to people in the transitional phase.)

Thus, we now have three kinds of people. There are knights, who only make true statements and knaves, who only make false statements. There are also people in the transitional phase. They are called "Neutrals" and only make statements with the truth value N.

The island's philosophers now needed to decide how to handle compound statements. Last week we saw how they handled if-then statements. This week we shall consider conjunctions ("and" statements) and disjunctions ("or" statements). The generally accepted truth tables are below:

$\wedge$	Т	Ν	F	_	V	Т	Ν	F
Т	Т	Ν	F		Т	Т	Т	Т
Ν	Ν	Ν	F		Ν	Т	Ν	Ν
F	F	F	F		F	Т	Ν	F

This table should be read as follows: If your statement has the form, "P and Q," then the truth of P is represented down the side of the table on the left and the truth of Q is represented across the top. Likewise for "P or Q," except that you should use the table on the right. In filling in these tables, our goal was to preserve as much of our classical intuition as possible. So, in those places where only classical truth values are used (specifically, the entries in the corners of the tables), everything is treated as though we were using classical logic. In filling in the entries where either P or Q is neutral, we use the principle that "and" statements are false as soon as either part is false, while "or" statements are true as soon as either part as true. If you interpret the truth value N as representing "possibly true and possibly false," then the tables practically fill in themselves.

With that in mind, what can you deduce from the following dialog?

You meet three people, who make the following statements:

Doctor Daystrom : Frankenstein is a knight and Frankenstein is a knave. Elmer Fudd : Frankenstein is a knave or Frankenstein is not a knave. Frankenstein : Doctor Daystrom is a knight or Elmer Fudd is a knave or I am a neutral.

Please determine the types of all three people.

Good luck with that! Everything may seem a little weird at first, but if you give it a try you might find that this is not quite as complex as it seems. Feel free to stop by my office or to send me an e-mail if you have questions.

Solutions are due to Jason Rosenhouse by 5:00 on Friday, April 11. Please write your solution clearly in the space below. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the upper right corner of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Please make sure that the answer to the problem is displayed clearly and prominently. Keep in mind, however, that to be considered correct, your answer to the problem must be accompanied by a clear, concise explanation that proves that your answer is the only one possible. Problems are available at the bulletin board outside Roop 119, and also at the website:

http://educ.jmu.edu/~rosenhjd/POTW/Spring14/homepage.html