Problem of the Week Solution Two

Recall that knights only make true statements while knaves only make false statements. Normals, on the other hand, sometimes make true statements and sometimes make false statements. Just like normal people! For residents of the island, it was customary to say that knights were of "higher rank" than normals, and that normals, in turn, were of "higher rank" than knaves. That's the set-up. Here's the problem:

You meet three people, whose names are Data, Ezri, and Fell. One of them is a knight, one is a knave, and one is a normal. Data says, "Ezri is of higher rank than Fell." Ezri then says, "Fell is of higher rank than Data." If you now ask Fell, "Who has higher rank, Data or Ezri?" how will Fell reply?

SOLUTION: Fell will say that Ezri is of higher rank than Data.

Start with Data's statement. If Data is a knight, then it is true that Ezri is of higher rank than Fell. That would imply that Ezri is a normal and Fell is a knave (since we know there is exactly one person of each type among the three.) On the other hand, if Data is a knave then it is false that Ezri is of higher rank than Fell. That would imply that Fell is a knight and Ezri is a normal. In neither case is Fell a normal. Moreover, if Data is a normal, then Fell certainly is not a normal (again, since there is only one person of each type.) We conclude that it is impossible for Fell to be a normal.

By essentially the same reasoning, we conclude from Ezri's statement that Data is not a normal. It follows that Ezri must be the normal.

Now let us consider how Fell will reply to our question. We know he is not a normal. If he is a knight, then Data must be a knave. In this case Ezri is of higher rank than Data, and Fell will truthfully assert that to be the case. If Fell is a knave, then Data must be the knight. In this case Data is of higher rank than Ezri, but Fell will lie and deny that that is the case. Either way, Fell will assert that Ezri is of higher rank than Data and we are done.