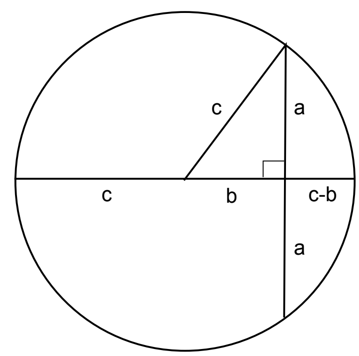
Problem of the Week
Number Three
February 9, 2015

Suppose you have a right triangle whose legs are a and b and whose hypotenuse is c. If you draw a circle with radius c and center at one of the non-right angle vertices of the triangle, the result will be something like the diagram to the right. I have taken the liberty of extending the side of length a. Since a diameter that is perpendicular to a chord bisects that chord (you should prove that as an exercise), we now have two segments of length a. It is also straightforward to show that the other segments have the lengths I have indicated in the diagram.



Now, I'm sure you remember the two-chord power theorem. That's the one that says that given two intersecting chords, the product of the two pieces of one of the chords equals the product of the two pieces of the other. (You should prove *that* as an exercise too. It's a similar triangles thing.) In this case, one of the chords is divided into two pieces of length a, while the other is cut into pieces of length $c + b$ and $c - b$. Applying the theorem now gives

$$a^2 = (c + b)(c - b) = c^2 - b^2.$$

That, my friends, is known as the Pythagorean theorem. It's a clever proof, don't you think? You may find the theorem helpful in handling this week's problem:

An isosceles triangle has a perimeter of 100.
The altitude drawn from the common vertex of the two congruent sides has length 30.
Find the area of the triangle.

Submissions are due to Jason Rosenhouse by 5:00 on Friday, February 13. ***Solutions should be written on the back of an official POTW handout.*** *Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks.* ***To be considered correct, your answer to the problem must be accompanied by a clear, concise explanation.*** *Solutions will be posted at this website, by the Monday after the problem is due:*