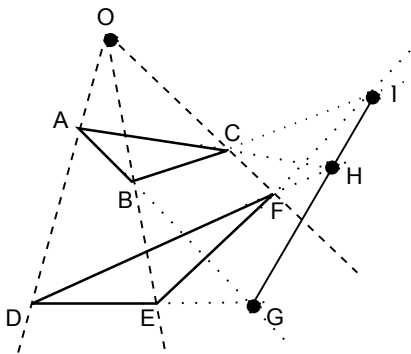

Problem of the Week

Number Seven

March 23, 2015

This week's problem involves triangles, so how about we look at one of the coolest triangle theorems there is?

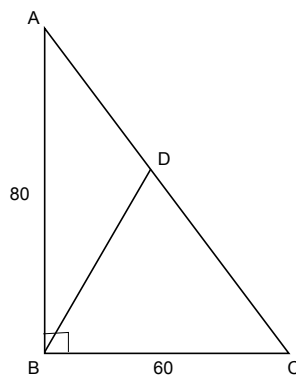
The diagram below may look very complicated, but it depicts something very cool. We start with triangles ABC and DEF . They have a very intriguing property. The lines joining pairs of vertices (dashed, in the figure); A to D , B to E , and C to F ; all meet at a point. I have labeled that point O . So far so good.



Now let us extend pairs of corresponding sides (the dotted lines in the figure). We find that sides AB and DE intersect at G . We also find that AC meets DF at H , and that BC meets EF at I . The cool part is that G , H , and I all lie on a straight line. And when I say that is cool, I mean that it is very cool indeed! It turns out this is always true. If the lines joining corresponding vertices of two triangles intersect at a point, then the points of intersection of the extensions of corresponding sides lie on a straight line. This is known as Desargue's theorem.

Even more interesting is that, while this may appear to be a straightforward result in plane geometry, its proof requires the introduction of a point outside the plane. This sort of technique is standard in projective geometry, but exploring that fact would take us beyond the scope of this POTW. So let's get on with the problem:

As shown to the right, ABC is a right triangle whose legs have length 60 and 80, and whose right angle is at B . Segment BD is drawn to the hypotenuse in such a way that triangles ABD and BCD have the same perimeter. Find the length of segment BD .



A challenging problem, I think, but see what you can come up with. Then follow the directions on the other side of this page. \implies

*Submissions are due to Jason Rosenhouse by 5:00 on Friday, March 27. **Solutions should be written on the back of an official POTW handout.** Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the **upper right corner** of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. **To be considered correct, your answer to the problem must be accompanied by a clear, concise explanation.** Solutions will be posted at this website, by the Monday after the problem is due:*

<http://educ.jmu.edu/~rosenhjd/POTW/Spring15/homepage.html>