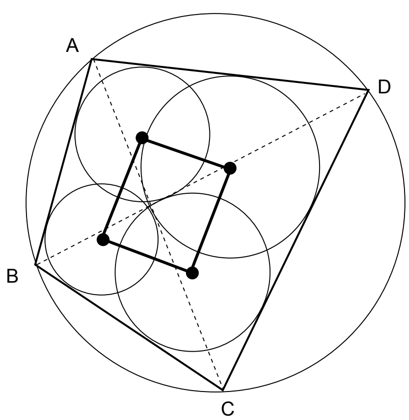
Problem of the Week
Number Eight
March 30, 2015

This week's problem involves a circle inscribed in a polygon. So how about I show you a cool theorem that involves circles inscribed in polygons?

You might have to stare at this diagram for a while before it is clear what is going on. Have a go at it, because it's worth the effort. We start with quadrilateral $ABCD$. It is a *cyclic quadrilateral*, which means simply that it can be inscribed in a circle. We then draw the two diagonals, dashed in the figure.



By drawing these diagonals, we have produced the four triangles ABC, ABD, ACD, and BCD. (In each triangle, two sides are the sides of the quadrilateral, while the third is a diagonal.) We then draw the *incircles* of the four triangles, the four small circles in the diagram. Each circle is tangent to the three sides of the triangle that contain it.

Now we're ready for the punchline. Identify the centers of the four circles, the heavy dots in the figure. Those four centers from the vertices of a rectangle. It works every time!

That's enough of that. Here's this week's problem:

A quadrilateral which has an inscribed circle has three consecutive sides of length 4, 9 and 16 respectively. Find the length of the fourth side.

Keep in mind that when we speak of a circle being inscribed in a quadrilateral, we mean that the circle is inside the quadrilateral and is tangent to all four sides. By "consecutive sides" we mean that the side of length 9 shares one of its corners with the side of length 4, and its other corner with the side of length 16. So mull that over, and when you think you have the answer follow the directions on the other side of this page. $\Longrightarrow$

Submissions are due to Jason Rosenhouse by 5:00 on Friday, April 3. ***Solutions should be written on the back of an official POTW handout.*** *Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks.* ***To be considered correct, your answer to the problem must be accompanied by a clear, concise explanation.*** *Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring15/homepage.html