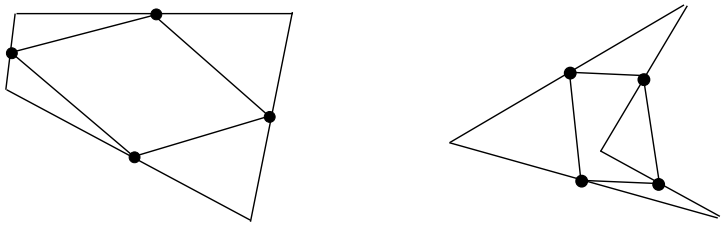
Problem of the Week
Number Nine
April 6, 2015

We have seen several theorems that have the same general form. A simple geometric object is constructed. From that object certain natural points are identified. It then turns out that those points arrange themselves in aesthetically pleasing ways.

In POTW 5 we saw that in any triangle, nine easily identified points arrange themselves in a circle. In POTW 7 we saw Desargues' theorem, which showed that two triangles that are in perspective from a point are in perspective from a line. In POTW 8 we saw an elaborate variation on the theme, which started with a cyclic quadrilateral and ended with four points magically defining a rectangle. It occurs to me, though, that I have not yet shown you one of the most famous theorems along these lines. It is easier to understand than the others, but that just makes it all the more shocking.

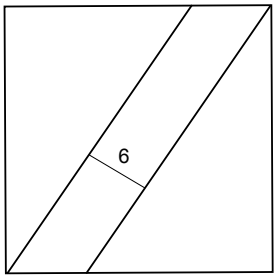
Take any quadrilateral. *Any* quadrilateral. Locate the midpoints of the sides. Connect the midpoints of adjacent sides. You have just drawn a parallelogram. Boom!



The figure on the left shows the basic scheme. The figure on the right shows that you do not even need to assume your quadrilateral is convex. When I said "any quadrilateral" I meant it!

As it happens, this week's problem also involves a parallelogram. See what you can do with this:

A square is divided into three regions by two parallel cuts, as shown to the right. The three regions have the same area, and the distance between the parallel lines is 6. Find the area of the square.



Mull that over, then meet me on the flipside $\Longrightarrow$

Submissions are due to Jason Rosenhouse by 5:00 on Friday, March 27. ***Solutions should be written on the back of an official POTW handout.*** *Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks.* ***To be considered correct, your answer to the problem must be accompanied by a clear, concise explanation.*** *Solutions will be posted at this website, by the Monday after the problem is due:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring15/homepage.html