

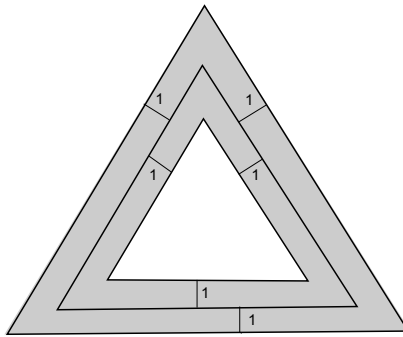
---

## Problem of the Week

### Solution Ten

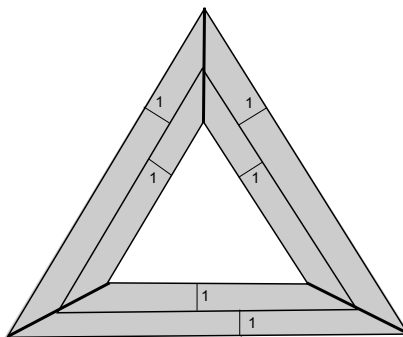
---

Below are three concentric triangles. The lengths of the sides of the middle triangle are 9, 10 and 11. Also, the distance between the sides of the middle triangle and each of the other two triangles is 1, as shown in the diagram. Find the area of the shaded region.



SOLUTION: The area is 60.

Draw in the three thick lines as shown below:



The shaded region has now been divided into three trapezoids. The area of a trapezoid is found by taking the average of the bases and multiplying by the height. Now, it is a basic fact about trapezoids that the length of its median, by which I mean the line segment joining the midpoints of the non-parallel sides, is equal to the average of the bases. (You might enjoy proving this for yourself!) It follows that the area of a trapezoid can be expressed as the product of the median and the height.

The three medians are precisely the lengths of the sides of the middle triangle. Moreover, the height of each trapezoid is easily seen to be 2. We conclude that the area of the shaded region is  $2(9 + 10 + 11) = 60$ .

As for the two warm-ups, in this fraction:

$$\frac{AEFHJKLMNTVWXY}{BCDGJOPQRSU}$$

the Z should go above the line. The letters above the line are the ones that can be made with straight lines only. The letters below require curves.

As for this sequence:

8 5 4 9 1 7 6 3

I recommend finishing it with 2 0. That way, if we write out the words instead of the digits, like this:

eight, five, four, nine, one, seven, six, three, two, zero

we find that the whole sequence is in alphabetical order.

See you next term!