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## Problem of the Week

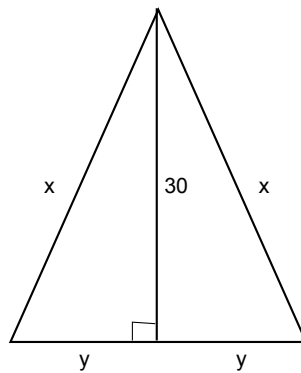
### Solution Three

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**An isosceles triangle has a perimeter of 100. The altitude drawn from the common vertex of the two congruent sides has length 30. Find the area of this triangle.**

SOLUTION: The area is 480.

It follows from the hypotenuse-leg theorem that the given altitude bisects the triangle. Let  $x$  denote the length of one of the congruent sides, and let  $y$  denote half the length of the base. The picture looks like this:



We can now produce the following system of two equations in two unknowns:

$$\begin{aligned}x + y &= 50 \\x^2 - y^2 &= 900\end{aligned}$$

The first equation follows from what we are given about the triangle's perimeter. The second equation follows from the Pythagorean theorem.

To solve the system, we manipulate the second equation as follows:

$$x^2 - y^2 = (x + y)(x - y) = 50(x - y) = 900.$$

It follows that  $x - y = 18$ . Adding this to the equation  $x + y = 50$  gives us  $2x = 68$  and  $x = 34$ . This immediately implies that  $y = 16$ . Keep in mind that  $y$  represents exactly half of the base.

Since the area of a triangle is  $\frac{1}{2}bh$ , we get an area of  $(30)(16) = 480$  in this case.