## Problem of the Week Solution Four

A piece of wire 40 inches long is cut into two pieces. One piece is formed into a square, while the other piece is formed into a circle that circumscribes the square. Determine the length of the shorter piece.

SOLUTION: The length of the shorter piece is  $\frac{160}{\pi\sqrt{2}+4}$ .

Denote the diameter of the circle by d, and denote the length of a side of the square by x, as shown below:



By the Pythagorean theorem, we have  $d = x\sqrt{2}$ . By calculating the circumference of the circle (using the formula  $C = \pi d$ ) and the perimeter of the square, we find that

$$x\pi\sqrt{2} + 4x = 40.$$

It follows that:

$$x = \frac{40}{\pi\sqrt{2}+4}$$

Since the perimeter of the square is plainly smaller than the circumference of the circle, the answer to the problem will be the perimeter of the square, which is 4x. The answer now follows immediately.