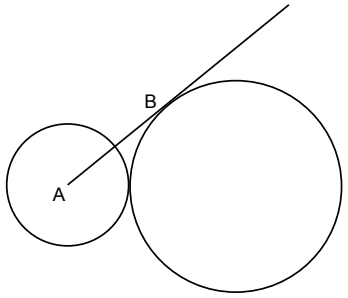
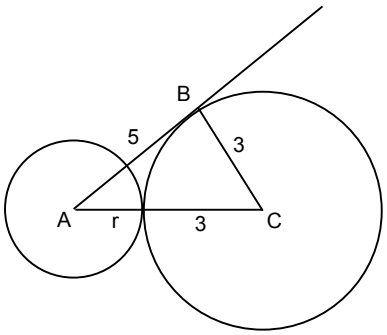
Problem of the Week
Solution Five

Two circles are externally tangent, as shown below. The smaller circle has its center at A. A line segment is drawn from A, tangent to the second circle at B. If AB has length 5, and the radius of the larger circle is 3, find the radius of the smaller circle.



SOLUTION: The radius of the smaller circle is $\sqrt{34} - 3$.

Let C denote the center of the larger circle, and draw radius BC. Label the radius of the smaller circle by r. Then our diagram, with the information given in the problem statement, now looks like this:



We must now use the theorem that says that a tangent to a circle is perpendicular to the radius drawn to the point of tangency. Thus, ABC is a right triangle. We can apply the Pythagorean theorem to write:

$$\begin{aligned} 5^2 + 3^2 &= (r+3)^2 \\ 34 &= r^2 + 6r + 9 \\ 0 &= r^2 + 6r - 25 \end{aligned}$$

The quadratic formula now shows that

$$x = \frac{-6 \pm \sqrt{36 + 100}}{2}.$$

Since a negative answer makes no sense in this context, we obtain the answer

$$x = \frac{-6 + \sqrt{136}}{2} = \frac{-6 + 2\sqrt{34}}{2} = \sqrt{34} - 3.$$