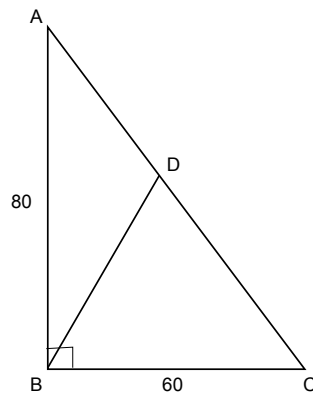

Problem of the Week

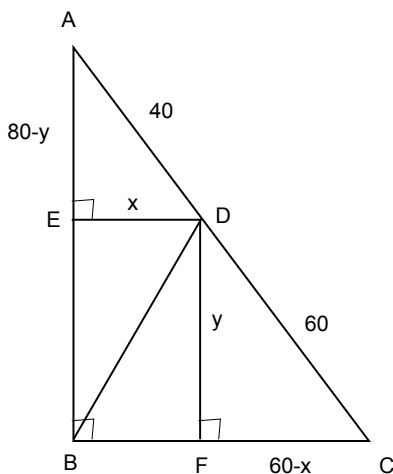
Solution Seven

ABC is a right triangle whose legs have length 60 and 80, and whose right angle is at *B*. Segment *BD* is drawn to the hypotenuse in such a way that triangles *ABD* and *BCD* have the same perimeter. Find the length of segment *BD*.



SOLUTION: Segment *BD* has length $24\sqrt{5}$.

Since the legs are 60 and 80, the Pythagorean theorem shows that the hypotenuse is 100. Since triangles *ABD* and *BCD* have the same perimeter, and since *BD* is shared by both triangles, we quickly see that *D* splits the hypotenuse into segments of length 40 and 60. Now draw segments *ED* and *DF* perpendicular to *AB* and *BC* respectively, and label the lengths of the various line segments as shown in the diagram:



Notice that triangles *AED* and *DFC* are both similar to triangle *ABC*. Since the corresponding sides are therefore proportionate, we can write the equations

$$\frac{x}{40} = \frac{60}{100} \quad \text{and} \quad \frac{y}{60} = \frac{80}{100}.$$

The first equation gives us that $x = 24$, while the second gives us that $y = 48$. One more application of the Pythagorean theorem shows that the length of *BD* is given by

$$\sqrt{x^2 + y^2} = \sqrt{(24)^2 + (48)^2} = 24\sqrt{5}.$$