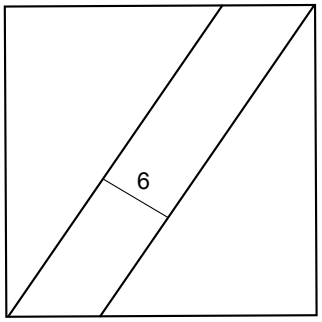
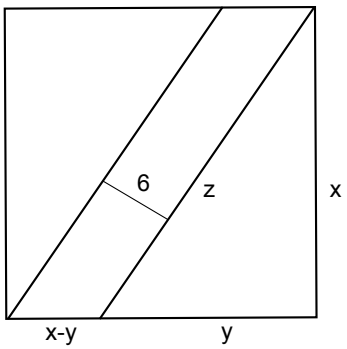
Problem of the Week
Solution Nine

A square is divided into three regions by two parallel cuts, as shown below. The three regions have the same area, and the distance between the parallel lines is 6. Find the area of the square.



SOLUTION: The area of the square is 468.

Label the sides of the figure as follows:



Since the area of the square is three times the area of one of the right triangles, we can write the equation:

$$x^2 = 3\left(\frac{1}{2}xy\right) = \frac{3}{2}xy.$$

This implies that $y = \frac{2}{3}x$.

Also, by the Pythagorean theorem we find that $z = \sqrt{x^2 + y^2}$.

The area of the square is also three times the area of the parallelogram. Recalling that the area of a parallelogram with base b and height h is given by $A = bh$, and using the z-side as the base, we see that the area of the parallelogram is $6z$. Thus, we can write:

$$x^2 = 18z = 18\sqrt{x^2 + y^2} = 18\sqrt{x^2 + \frac{4}{9}x^2} = 18\sqrt{\frac{13}{9}x^2} = 6x\sqrt{13}.$$

This implies that $x = 6\sqrt{13}$. It follows that the area of the square is $x^2 = (36)(13) = 468$, as claimed.