Problem of the Week Solution Two

PROBLEM: There are five positive integers strictly less than 6, and 3 of these are factors of 6 (specifically: 1, 2, and 3). That means that 60% of the positive integers strictly smaller than 6 are factors of 6. What is the smallest positive integer n for which fewer than 1% of the positive integers less than n are factors of n?

SOLUTION: The answer is 103. Since 1 is a factor of any number, and since we require that **fewer** than 1% of the positive integers strictly less than n be factors of n, we see that there must be more than 100 numbers strictly less than n. It follows that n > 101. Plainly, 102 will not work. So we try 103, which is the smallest prime number larger than 101, and we find that it fits the bill.