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# Problem of the Week

## Solution Four

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**PROBLEM:** Find the smallest, positive, integral value of  $n$  for which the fraction

$$\frac{n - 12}{5n + 23}$$

is both non-zero and reducible.

**SOLUTION:** The answer is  $n = 95$ .

For the fraction to be reducible, we need  $n - 12$  and  $5n + 23$  to have a common factor other than one. So, set  $x = n - 12$ . Then  $5n + 23 = 5x + 83$ . Our fraction now becomes

$$\frac{x}{5x + 83}$$

If  $x$  is such that it shares a common factor with  $5x + 83$ , then it must be that  $x$  is a multiple of 83. The smallest such multiple is 83 itself. So  $x = 83$ , from which it follows that  $n = 95$ .

An alternative solution is to first invert the fraction, giving us

$$\frac{5n + 23}{n - 12}$$

If we carry out the division, the result is

$$5 + \frac{83}{n - 12}$$

Our problem now reduces to that of finding the smallest value of  $n$  for which  $\frac{83}{n-12}$  is reducible. This will happen when 83 and  $n - 12$  share a common factor other than one. Since 83 is prime, the smallest value of  $n$  that works is the one for which  $n - 12 = 83$ . Once again we arrive at the answer  $n = 95$ .