
Problem of the Week

Solution Six

PROBLEM: *There are eight numbers that can be made with four two's and no other symbols. Here they are:*

$$\begin{array}{cccc} 2222 & 222^2 & 22^{22} & 2^{222} \\ 22^{22} & 2^{22^2} & 2^{2^{22}} & 2^{2^{2^2}} \end{array}$$

Which is the largest of these numbers? Keep in mind that an expression of the form a^{b^c} is understood to mean $a^{(b^c)}$.

SOLUTION: The seventh number, $2^{2^{22}}$, is the largest.

The trick here is to chip away at it. The number 2222 is clearly the smallest of the eight, so we discard it. To compare the next two we find that $22^{22} = (22^{11})^2$. Since 22^{11} is much bigger than 222, we discard the second number as well. But we also have that

$$22^{22} < 32^{22} = (2^5)^{22} = 2^{110}.$$

This number is plainly smaller than 2^{222} .

We conclude that 2^{222} is the largest of the first four numbers.

The fifth number is just 22^4 , while the last number is 2^{16} , and these numbers are clearly much smaller than others on the list.

Thus, the three remaining candidates are the fourth, sixth, and seventh numbers. Since all

of these are powers of two, we need only compare the exponents. We find that those exponents are, respectively, 222, 484, and 2^{22} . This final exponent is plainly the largest, from which our answer follows.