
Problem of the Week

Solution Seven

PROBLEM: *The product of two positive integers is 9984. The greatest common divisor of the two integers is equal to the difference between the two integers. Find the sum of the two integers.*

SOLUTION: The two numbers are 96 and 104. Their sum is 200.

First, notice that $9984 = (12)(13)(8^2)$.

Now, denote by d the greatest common divisor of the two integers. Then the smaller of the two integers can be expressed as kd , for some integer k . Since d also represents the difference between the two numbers, the larger of the two integers is $kd + d = (k + 1)d$.

We now have the equation

$$k(k + 1)d^2 = 9984.$$

The only way this is possible is to have $k = 12$ and $d = 8$. We conclude that the two numbers are $12 \times 8 = 96$ and $13 \times 8 = 104$. The sum of these integers is 200, as claimed.