Problem of the Week
Solution Eight

PROBLEM: *Let A be the set*

$$\{10, 11, 12, \ldots, 20\},$$

and let B be the set

$$\{21, 22, 23, \ldots, 30\}.$$

Each element of the first set is multiplied, in turn, by each element of the second set. Find the sum of all these products.

SOLUTION: The answer is $42{,}075$.

We are essentially being asked to evaluate the following expression:

$$\begin{aligned}
&10(21 + 22 + \cdots + 30) + \\
&11(21 + 22 + \cdots + 30) + \\
&12(21 + 22 + \cdots + 30) + \\
&\qquad\vdots \\
&20(21 + 22 + \cdots + 30)
\end{aligned}$$

But this is readily seen to be equivalent to:

$$(10 + 11 + \cdots + 20)(21 + 22 + \cdots + 30).$$

Evaluating each sum individually and then multiplying gives us

$$165 \times 255 = 42{,}075$$

as claimed.