
Problem of the Week

Solution Nine

PROBLEM: *A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of the position. (That is, it skips 4 regardless of whether the 4 is in the units digit, the tens digit, the hundreds digit, and so on.) For example, after traveling one mile the odometer changes from 39 to 50. If the odometer now reads 2016, how many miles has the car actually traveled?*

SOLUTION: The answer is 1472.

The simplest approach is to notice that since the odometer only uses nine digits, we are effectively counting in base nine. However, the odometer uses the digits 5, 6, 7, 8, and 9 when normally we would use 4, 5, 6, 7, and 8 when counting in base nine, respectively. That means the odometer reading of 2016 corresponds to a total mileage of 2015_9 . So we now compute:

$$\begin{aligned} 2015_9 &= (2 \times 9^3) + (0 \times 9^2) + (1 \times 9) + 5 \\ &= (2 \times 729) + (1 \times 9) + 5 = 1472, \end{aligned}$$

as claimed.