Problem of the Week
Number Eight
March 27, 2017

In his 1847 work *Mathematical Analysis of Logic*, George Boole showed how standard Aristotelian syllogisms could be interpreted algebraically. For example, consider the premises, "All cats are mammals," and "All mammals are animals." Let us take c to represent cats, m to represent mammals, and a to represent animals.

Then our two premises might be represented like this:

$$c(1-m)=0 \qquad m(1-a)=0.$$

The idea is that multiplication represents conjunction ("and"), while 0 represents falseness or nonexistence. The expressions $(1-m)$ and $(1-a)$ represent not being a mammal and not being an animal. Translated back into English, the first equation now says, "There is nothing that is both a cat and not a mammal," while the second says, "There is nothing that is both a mammal and not an animal."

Now let's do some manipulations. If we distribute in each of the first two equations, the result is:

$$c=cm \qquad m=ma.$$

Substituting ma for m now gives

$$c=cma.$$

Since $cm=c$, this equation simplifies to

$$c=ca.$$

It follows that

$$c-ca=0 \quad \text{and} \quad c(1-a)=0.$$

Translated back to English, this says simply that there is nothing that is a cat and not an animal. Of course, this is precisely the conclusion standard Aristotelian reasoning would tell us to expect. You could argue that this is a rather convoluted way of arriving at an obvious conclusion, and you would be right. But this is just the tip of a big iceberg, and it is rather elegant taken by itself.

Speaking of elegant, here is this week's problem:

Suppose I make the definition that $a \uparrow b$ is the same thing as a^b. Solve for x in the following equation:

$$\frac{4 \uparrow (3 \uparrow 2)}{(4 \uparrow 3) \uparrow 2} = 4 \uparrow (3 \uparrow x)$$

When you think you have the problem figured out, follow the instructions below.

Submissions are due to Jason Rosenhouse by 5:00 on ***Friday, March 31.*** *Solutions, complete with a brief explanation, should be written on the back of an official POTW handout. Place your name, e-mail address, and the section numbers and professors of any math courses you are taking, in the* ***upper right corner*** *of the front of the page. One weekly winner will receive a five-dollar gift card from Starbucks. Solutions will be posted at the POTW website:*

http://educ.jmu.edu/∼rosenhjd/POTW/Spring17/homepage.html