Problem of the Week Solution Two

If

$$\frac{1}{x^3} - \frac{1}{x^2} - \frac{1}{x} - 1 = 0,$$

then what is the value of

$$x^3 + x^2 + x + 2?$$

SOLUTION: The answer is 3.

We are given that

$$\frac{1}{x^3} - \frac{1}{x^2} - \frac{1}{x} - 1 = 0.$$

If we multiply both sides by $-x^3$ the result is

$$x^3 + x^2 + x - 1 = 0.$$

If we now add 3 to both sides of this equation, the result is

$$x^3 + x^2 + x + 2 = 3,$$

as claimed.