
Problem of the Week

Solution Five

Find all ordered triples (x, y, z) that satisfy the system

$$(x + y)(x + y + z) = 120$$

$$(y + z)(x + y + z) = 96$$

$$(x + z)(x + y + z) = 72$$

SOLUTION: The only triples that work are $(4, 6, 2)$ and $(-4, -6, -2)$.

The key idea here is to add the three equations together. The result is

$$(2x + 2y + 2z)(x + y + z) = 288.$$

Dividing both sides by 2 leaves us with

$$(x + y + z)^2 = 144,$$

which implies that $x + y + z = \pm 12$.

Let's work with the positive solution first. Our system of equations now reduces to

$$x + y = 10$$

$$y + z = 8$$

$$x + z = 6$$

This system can be solved in a variety of ways, but perhaps the simplest is to subtract each equation from $x + y + z = 12$. Doing this leads us immediately to $x = 4$, $y = 6$ and $z = 2$.

Had we used $x + y + z = -12$, the same reasoning would have led us to the solution $x = -4$, $y = -6$ and $z = -2$.