Problem of the Week
Solution Six
March 13, 2017

Find all ordered pairs of positive integers that satisfy the equation

$$xy + 6x - 11 = x^2.$$

As always, your answer does not count unless it comes with an explanation for how you know you found all the solutions.

SOLUTION: The only solutions are $(1, 6)$ and $(11, 6)$.

The key idea is to get all the terms involving x on one side, and the other terms on the other. The result is:

$$xy + 6x - x^2 = 11.$$

We factor to get

$$x(y + 6 - x) = 11.$$

Since we are given that x and y are positive integers, we find that $x = 1$ or $x = 11$. The first case leads to $y = 6$. The second case also leads to $y = 6$.