
Problem of the Week

Solution Seven

Suppose a certain parallelogram has three of its vertices at the points $(0, 0)$, $(3, 0)$ and $(4, 4)$. Find all three of the possibilities for the location of the fourth vertex.

SOLUTION: The three possibilities are $(-1, -4)$, $(1, 4)$, and $(7, 4)$.

Here is one method for arriving at this solution. Recall that the diagonals of a parallelogram have the same midpoint. That is, the diagonals cross at a point that bisects each of the diagonals. Now, recall that the midpoint of the line segment connecting the point (a, b) to the point (c, d) is found by taking the average of the x -coordinates, and the average of the y -coordinates, like this:

$$\left(\frac{a + c}{2}, \frac{b + d}{2} \right).$$

Applied to the present context, four points can represent the vertices of a parallelogram only if the x -coordinate sums of opposite points are the same, and if the y -coordinate sums of opposite points are also the same.

So, if $(0, 0)$ and $(3, 0)$ are taken to be opposites, then the x sum is three and the y sum is 0. Thus, the point that gets paired with $(4, 4)$ must be $(-1, -4)$.

If $(0, 0)$ and $(4, 4)$ are opposites, then the x sum is 4 and the y sum is also 4. Thus, the point that gets paired with $(3, 0)$ must be $(1, 4)$.

Finally, if $(3, 0)$ and $(4, 4)$ are opposites, then the x sum is 7 and the y sum is 4. Thus, the point that gets paired with $(0, 0)$ must be $(7, 4)$.