Department of Mathematics and Statistics Colloquium

Bootstrap Percolation in the Plane

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Abstract: Bootstrap percolation is a natural growth process arising in the study of ferromagnetic structures; this process has found great application within statistical physics, epidemiology, the study of fault tolerances, and numerous other fields. In addition, it has become a favorite model of both combinatorialists and probabilists.

The process is simple; in order to define 'k-neighbor bootstrap percolation' all we need is a subset of the plane, X, and an integer k. We start by partitioning X into polygons, and initially 'infecting' some of the polygons. An 'uninfected' polygon becomes infected when at least k of its neighbors are infected. Once infected, a polygon is doomed – it will never become infected.

Does the infection spread to the entire set? Does it stop? How long might the infection take to spread? How quickly could it happen? These are all natural questions, whose answers are unknown in general. In this talk, we'll focus on the case where our ground set is the entire plane, and polygons are initially infected at random. Even in this simple case, analysis is tricky and there are surprising theorems to be found.

Monday, October 8 at 3:50 in Roop 103

Refreshments at 3:30