MATH 103 Sochacki Turn In Assignment 1
Due May 27, 2015 50 Points

Name KEY WEST

Work on the following problems. Your work must be written neatly on 8.5x11 inch paper with this sheet on the top of your write up or I will not grade your work. All necessary work must be shown for credit. Your work must represent the question asked. You may discuss this assignment with others, but all work turned in must be your own work. Your work is more important than the answer.

I have neither received nor given help on this project. Mon Key
(Signature)

1. Give sets of cardinality 2, 3 and 4 that contain words that are spelled similarly, but are pronounced differently.
2. Give the two forms of Pascal's triangle that contains 11 rows. List five DIFFERENT pattern rules that occur in these triangles.
3. Give the first 10 Fibonacci numbers. Give the ratio $\frac{F_{n+1}}{F_n}$ for these numbers. What is the cardinality of this ratio set? How many sets of cardinality five can be made from this ratio set?
4. Give a set that has cardinality four. Give all the subsets that can be made from this set.
5. Give the value of the following. (a) C_3^9 (b) C_6^9 (c) $\sum_{i=1}^{5}(4i-3)$ (d) $\sum_{i=1}^{8}3^i$ (e) $\sum_{i=0}^{8}\left(\frac{1}{2}\right)^i$
6. Give a formula or rule for each of the following that works for any $n \in N$.
   (a) $C_m^n = C_?^n$ (b) $\sum_{i=1}^{n}(4i-3) = ?$ (c) $\sum_{i=0}^{n}C_i^n = ?$ (d) $\sum_{i=1}^{n}\left(\frac{3}{4}\right)^i = ?$ (e) $\sum_{i=1}^{n}4i = ?$
7. A boy has 8 flags numbered 1,2,3,4,5,6,7,8. How many different 3 flag signals can he make with these 8 flags? How many different 3 flag sets can he make with these 8 flags? How many different flag sets can he make with these 8 flags?
8. You have 6 quarters. How many different outcomes are possible if you flip these 6 quarters? How many of these outcomes have 2 tails? How many of these outcomes have 3 tails? How many of these outcomes have 4 tails?

0	1												
1	1	1											
2	1	2	1										
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	10	10	5	1							
6	1	6	15	20	15	6	1						
7	1	7	21	35	35	21	7	1					
8	1	8	28	56	70	56	28	8	1				
9	1	9	36	84	126	126	84	36	9	1			
10	1	10	45	120	210	252	210	120	45	10	1		
11	1	11	55	165	330	462	462	330	165	55	11	1	
12	1	12	66	220	495	792	924	792	495	220	66	12	1

3. $S = \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55\}$

$R = \left\{ \frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34} \right\}$

$= \{1, 2, 1.5, 1.\overline{6}, 1.6, 1.625, 1.\overline{615384}, 1.\overline{619047}, 1.617\ldots\}$

$|R| = 9$

$C_5^9 = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 9 \cdot 2 \cdot 7 = 126$

4. $S = \{a, b, c, d\}$ S has $2^4 = 16$ subsets

$C_0^4 = 1$: φ $C_1^4 = 4$: $\{a\}, \{b\}, \{c\}, \{d\}$

$C_2^4 = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6$: $\{a, b\}, \{a, c\}, \{a, d\}$ $\{b, c\}, \{b; d\}$ $\{c, d\}$

$C_3^4 = \frac{4!}{3!1!} = 4$: $\{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

$C_4^4 = 1$: $\{a, b, c, d\}$

5. (a) $C_3^9 = \frac{9!}{3!6!} = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84 = C_6^9$ (b)

(c) $\sum_{i=1}^{5} (4i - 3) = 1 + 5 + 9 + 13 + 17 = \frac{5(1+17)}{2} = 45$

(d) $\sum_{i=1}^{8} 3^i = 3 + 3^2 + 3^3 + \cdots + 3^8 = \sum_{i=0}^{8} 3^i - 1$

$= \frac{3^9 - 1}{3 - 1} = \frac{19683 - 1}{2}$

(e) $\sum_{i=0}^{8} \left(\frac{1}{2}\right)^i = \frac{\left(\frac{1}{2}\right)^9 - 1}{\frac{1}{2} - 1} = \frac{\frac{1}{512} - 1}{-\frac{1}{2}} = 2\left(1 - \frac{1}{512}\right) = 2\frac{511}{512}$

$= \frac{511}{256}$

6. (a) $C_m^n = C_{n-m}^n$

(b) $\sum_{i=1}^{n}(4i-3) = \frac{n(1+4n-3)}{2} = \frac{n(4n-2)}{2} = \frac{n\,2(2n-1)}{2}$

$= n(2n-1)$

(c) $\sum_{i=0}^{n} C_i^n = 2^n$

(d) $\sum_{i=1}^{n}\left(\frac{3}{4}\right)^i = \sum_{i=0}^{n}\left(\frac{3}{4}\right)^i - 1$

$= \frac{\left(\frac{3}{4}\right)^{n+1} - 1}{\frac{3}{4} - 1} - 1$

$= \frac{1 - \left(\frac{3}{4}\right)^{n+1}}{\frac{1}{4}} - 1$

$= 4\left(1 - \left(\frac{3}{4}\right)^{n+1}\right) - 1$

$= 3 - 4\left(\frac{3}{4}\right)^{n+1}$

(e) $\sum_{i=1}^{n} 4i = 4\sum_{i=1}^{n} i = 4\,\frac{n(n+1)}{2} = 2n(n+1)$

7. $\underline{8}\ \underline{7}\ \underline{6}$ $\quad 8 \cdot 7 \cdot 6$

$$C_3^8 = \frac{8!}{3!\,5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

2^8 is the total number of sets

8. $\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$

2^6 total outcomes

$$C_2^6 = \frac{6!}{2!\,4!} = \frac{6 \cdot 5}{2 \cdot 1} = 15 \quad \text{have 2 tails}$$

$$C_3^6 = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \quad \text{have 3 tails}$$

$$C_4^6 = C_2^6 = 15 \quad \text{have 4 tails}$$