MATH 103 Sochacki Turn In Assignment 2
Due May 28, 2015 50 Points

Name KEY WEST

Work on the following problems. Your work must be written neatly on 8.5x11 inch paper with this sheet on the top of your write up or I will not grade your work. All necessary work must be shown for credit. Your work must represent the question asked. You may discuss this assignment with others, but all work turned in must be your own work. Your work is more important than the answer.

I have neither received nor given help on this project. Don Key
(Signature)

1. You flip 12 coins. How many of the outcomes will have exactly 4 heads? How many of the outcomes will have exactly 8 tails? How many outcomes will have at most 5 heads? How many outcomes will have at least 1 tail?

2. Give the value as a ratio for the following. (a) $\sum_{i=2}^{12} C_i^{12}$

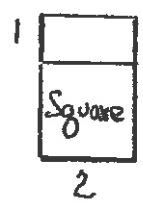
   (b) $\sum_{k=8}^{120} (5k-3)$

   (c) $\sum_{n=5}^{34} \left(\frac{3}{4}\right)^n$

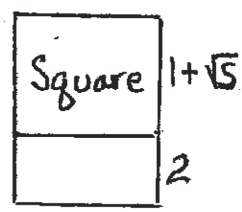
   (d) 0.137137137…

3. Determine which of the following rectangles below are golden rectangles.

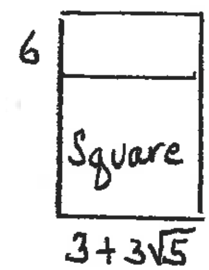
(a)



(b)

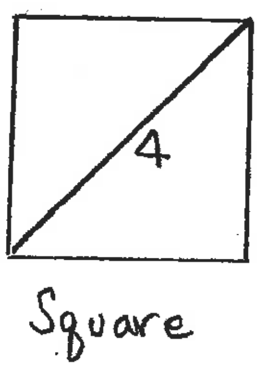


(c)

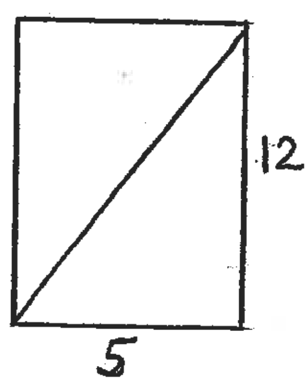


4. Give the lengths of all the line segments in each of the figures below.

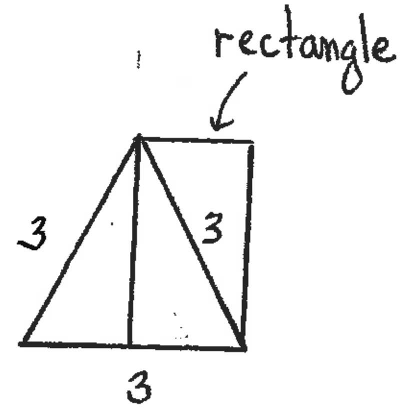
(a)



(b)



(c)



5. Give the approximation to $\sqrt{5}$ as a ratio given by the Fibonacci continued fraction from using 9 ones.

1. $C_4^{12} = \frac{12!}{4!\,8!} = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2\cdot 1} = 11\cdot 5\cdot 9 = 495$
have 4 heads

$C_8^{12} = C_4^{12}$ have 4 tails

$C_0^{12} + C_1^{12} + C_2^{12} + C_3^{12} + C_4^{12} + C_5^{12}$
is 5 heads or less

$C_1^{12} + \cdots + C_{12}^{12} = \sum_{i=1}^{12} C_i^{12}$ is at least one head

$= 2^{12} - C_0^{12} = 2^{12} - 1 = 4096 - 1 = 4095$

2. (a) $\sum_{i=2}^{12} C_i^{12} = \sum_{i=0}^{12} C_i^{12} - C_0^{12} - C_1^{12} = 2^{12} - 1 - 12$

(b) $\sum_{K=8}^{120} (5K-3) = 37 + 42 + \cdots + 600 - 3$

$= 37 + 42 + \cdots + 597$

$= \frac{113\,(37+597)}{2} = 35{,}821$

(c) $\sum_{n=5}^{34} \left(\frac{3}{4}\right)^n = \sum_{n=0}^{34} \left(\frac{3}{4}\right)^n - \sum_{n=0}^{4} \left(\frac{3}{4}\right)^n$

$= \frac{\left(\frac{3}{4}\right)^{35} - 1}{\frac{3}{4} - 1} - \frac{\left(\frac{3}{4}\right)^{5} - 1}{\frac{3}{4} - 1}$

$= \frac{\left(\frac{3}{4}\right)^{35} - \left(\frac{3}{4}\right)^{5}}{-\frac{1}{4}} = 4\left(\left(\frac{3}{4}\right)^{5} - \left(\frac{3}{4}\right)^{35}\right)$

(d) $0.137137137\ldots = 0.\overline{137} = \dfrac{137}{999}$

3 (a) $\dfrac{2+1}{2} \neq \dfrac{2}{1}$ NO

(b) $\dfrac{3+\sqrt{5}}{1+\sqrt{5}} = \dfrac{1+\sqrt{5}}{2}$

$$2(3+\sqrt{5}) = (1+\sqrt{5})^2$$

$$= 1+5+2\sqrt{5}$$

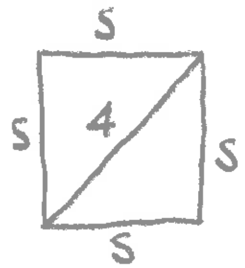
$$= 6+2\sqrt{5}$$ YES

(c) $\dfrac{9+3\sqrt{5}}{3+3\sqrt{5}} = \dfrac{3+3\sqrt{5}}{6}$

$$\frac{3(3+\sqrt{5})}{3(1+\sqrt{5})} = \frac{3(1+\sqrt{5})}{6}$$

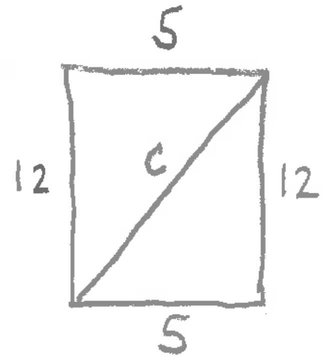
$\dfrac{3+\sqrt{5}}{1+\sqrt{5}} = \dfrac{1+\sqrt{5}}{2}$ same as (a)

4 (a)



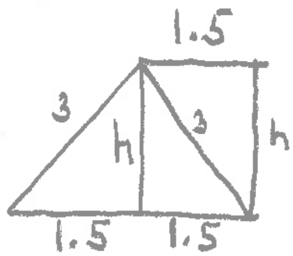
$$s^2 + s^2 = 4^2$$
$$2s^2 = 16$$
$$s^2 = 8, \quad s = \sqrt{8}$$

(b)



$$c^2 = 5^2 + 12^2$$
$$= 25 + 144 = 169$$
$$= 13^2$$
$$c = 13$$

(d)



$$1.5^2 + h^2 = 3^2$$
$$h^2 = 3^2 - 1.5^2$$
$$h = \sqrt{9 - 2.25} = \sqrt{6.75}$$

5. $$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1}}}} \cdots = \varphi = \frac{1 + \sqrt{5}}{2}$$

$$2\varphi = 1 + \sqrt{5}$$
$$2\varphi - 1 = \sqrt{5}$$

$$\varphi \approx 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{1}}}} = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \frac{1}{2}}} = 1 + \cfrac{1}{1 + \frac{1}{\frac{3}{2}}}$$
$$= 1 + \cfrac{1}{1 + \frac{2}{3}}$$

$$= 1 + \frac{1}{\frac{5}{3}} = 1 + \frac{3}{5} = 1.6 \Rightarrow \sqrt{5} \approx 2(1.6) - 1 = 2.2$$