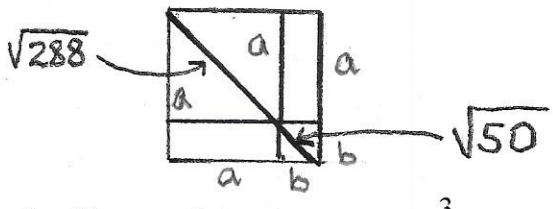
MATH 103 Sochacki Turn In Assignment 3
Due June 2, 2015 50 Points

Name Key West

Work on the following problems. Your work must be written neatly on 8.5x11 inch paper with this sheet on the top of your write up or I will not grade your work. All necessary work must be shown for credit. Your work must represent the question asked. You may discuss this assignment with others, but all work turned in must be your own work. Your work is more important than the answer.

I have neither received nor given help on this project. Tur. Key
(Signature)

1. Give the area of all the squares in the following figure.



$a^2+a^2=288$ $b^2+b^2=50$
$2a^2=288$ $2b^2=50$
$a^2=144$ $b^2=25$
$a=12$ $b=5$
$a+b=17$ $(a+b)^2=289$

2. Give a polynomial for (a) $\frac{3}{1-4x}$ (b) $\frac{1}{1-t-t^2}$ (c) $\frac{1+x}{1-x-x^2}$

3. Given the 8 spaces _ _ _ _ _ _ _ _, answer the following. How many different ways can you fill these spaces with the two numbers 0 and 1? How many of these have exactly three 1's? How many have exactly five 0's? How many have at least six 1's?

4. If you compute the sum $6+11+16+21+26+\ldots+501+506$, what natural number do you obtain?

5. Give a rational number for the following. (a) $0.\overline{1203}$ (b) $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\frac{1}{81}+\ldots.$ (c) $12.3\overline{478}$

6. Plot $x=3, x=\frac{7}{4}, x=\varphi, x=-\varphi, x=-\frac{7}{4}, x=-3$ all on the same real line. Also plot all the numbers satisfying $|x|=\frac{15}{4}$.

7. Give a real number for φ^8. Is this number rational or irrational?

8. Give a real number that the following continued fractions equal (represent).

(a) $1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{\ldots}}}}$ (b) $2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\ldots}}}}$

9. Solve the following for x. (a) $2x-5=7$ (b) $\frac{4x-3}{5}=1$ (c) $x^2-4x=5$ (d) $3x^2-2x+5=0$

2 (a) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

$\frac{3}{1-x} = 3 + 3x + 3x^2 + 3x^3 + \cdots$

$\frac{3}{1-t} = 3 + 3t + 3t^2 + 3t^3 + \cdots$

$\frac{3}{1-4x} = 3 + 3(4x) + 3(4x)^2 + 3(4x)^3 + \cdots$

$$
\begin{array}{r|l}
1-4x & 3 + 3(4x) + 3(4x)^2 + 3(4x)^3 + \cdots \\
\hline
 & 3 \\
 & \underline{3 - 3(4x)} \\
 & \quad 3(4x) \\
 & \quad \underline{3(4x) - 3(4x)^2} \\
 & \qquad 3(4x)^2 \\
 & \qquad \underline{3(4x)^2 - 3(4x)^3} \\
 & \qquad\quad 3(4x)^3
\end{array}
$$

(b) $\frac{1}{1-t-t^2}$

$$
\begin{array}{r|l}
1-t-t^2 & 1 + t + 2t^2 + 3t^3 + 5t^4 + 8t^5 + 13t^6 + 21t^7 \\
\hline
 & 1 \\
 & \underline{1 - t - t^2} \\
 & \quad t + t^2 \\
 & \quad \underline{t - t^2 - t^3} \\
 & \qquad 2t^2 + t^3 \\
 & \qquad \underline{2t^2 - 2t^3 - 2t^4} \\
 & \qquad\quad 3t^3 + 2t^4 \\
 & \qquad\quad \underline{3t^3 - 3t^4 - 3t^5} \\
 & \qquad\qquad 5t^4 + 3t^5
\end{array}
$$

2(c) $\dfrac{1+x}{1-x-x^2}$

$$
\begin{array}{r|l}
1-x-x^2 & 1+2x+3x^2+5x^3+8x^4+13x^5+21x^6+\cdots \\
\hline
1+x & \\
1-x-x^2 & \\
\hline
2x+x^2 & \\
2x-2x^2-2x^3 & \\
\hline
3x^2+2x^3 & \\
3x^2-3x^3-3x^4 & \\
\hline
5x^3-3x^4 & \\
5x^3-5x^4-5x^5 & \\
\hline
8x^4+5x^5 &
\end{array}
$$

3. $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$

2^8 total choices, each choice is a number

$$C_3^8 = \frac{8!}{3!5!} = \frac{8\cdot 7\cdot 6\cdot 5!}{3\cdot 2\cdot 1\cdot 5!} = 56$$ choices have exactly three 1's

$$C_5^8 = C_3^8 = 56$$

$$C_6^8 + C_7^8 + C_8^8 = \frac{8!}{2!6!} + \frac{8!}{7!1!} + 1$$

$$= 28 + 8 + 1 = 37$$

4. $6+11+16+21+26+\cdots+501+506$

$1 \quad 2 \quad 3 \quad 4 \quad 5 \qquad\qquad \underbrace{100 \quad 101}$

Count by 5's ←

$$\frac{101\cdot(506+6)}{2} = 101\cdot 256 = 25856$$

$$\sum_{K=1}^{101} 5K+1 = 6+11+16+\cdots+501+506$$

5. (a) $0.\overline{1203} = \frac{1203}{9999} = \frac{401}{3333}$

(b) $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\frac{1}{81}+\cdots$

$= 1+\left(\frac{-1}{3}\right)+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)^3+\cdots$

$= \dfrac{\left(-\frac{1}{3}\right)^{n+1}-1}{-\frac{1}{3}-1} \rightarrow \dfrac{-1}{-\frac{4}{3}} = \frac{3}{4}$

(c) $12.34\overline{78} = 12+\frac{34}{100}+\frac{78}{9900}$

$x = 12.34\overline{78}$

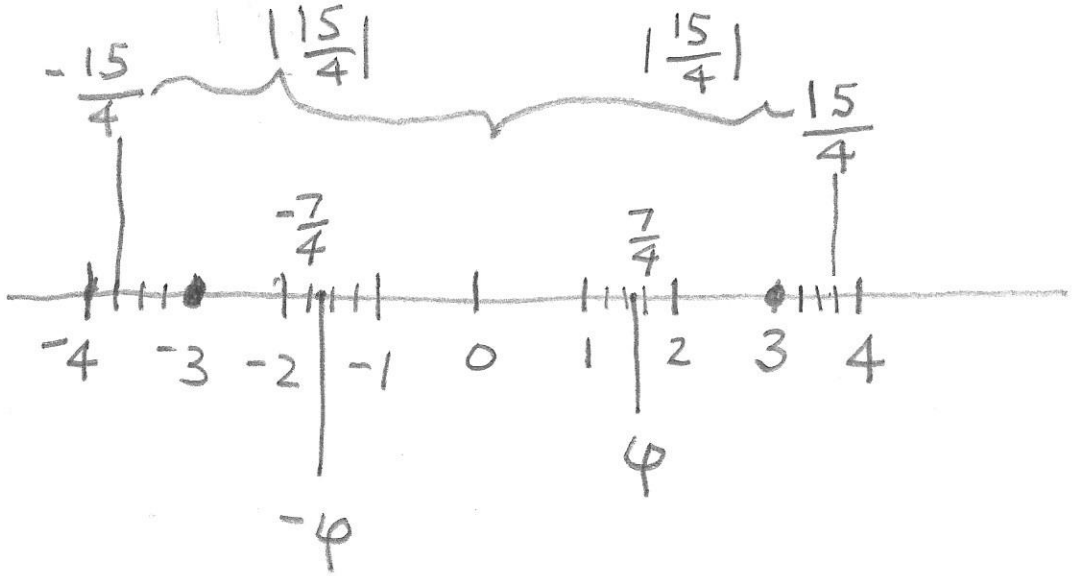
$100x = 1234.78\overline{78}$

$-\ x = 12.3478$

$99x = 1222.44$

$x = \frac{12244}{9900}$

6.



7. $\varphi^2 = \varphi + 1$

$\varphi^3 = \varphi^2 + \varphi = \varphi + 1 + \varphi = 2\varphi + 1$

$\varphi^4 = \varphi(2\varphi+1) = 2\varphi^2 + \varphi = 2(\varphi+1) + \varphi = 3\varphi + 2$

$\varphi^5 = \varphi(3\varphi+2) = 3\varphi^2 + 2\varphi = 3(\varphi+1) + 2\varphi = 5\varphi + 3$

$\varphi^6 = 8\varphi + 5$

$\varphi^7 = 13\varphi + 8$

$\varphi^8 = 21\varphi + 13 = 21\,\frac{1+\sqrt{5}}{2} + 13$

$= \frac{21 + 21\sqrt{5}}{2} + \frac{26}{2}$

$= \frac{47 + 21\sqrt{5}}{2}$

8(a) $$1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+2}}} = f$$

$$f = 1 + \frac{2}{f}$$

$$f^2 = f + 2$$

$$f^2 - f - 2 = 0 \Rightarrow f = \frac{1+\sqrt{1+4(2)}}{2} = 2$$

(b) $$2+\cfrac{1}{2+\cfrac{1}{2}} = F$$

$$F = 2 + \frac{1}{F}$$

$$F^2 = 2F + 1$$

$$F^2 - 2F - 1 = 0 \qquad F = \frac{2+\sqrt{4+4}}{2}$$

$$= \frac{2+\sqrt{8}}{2}$$

$$= \frac{2+2\sqrt{2}}{2}$$

$$= 1+\sqrt{2}$$

9 (a) $2x - 5 = 7$

$2x = 12$

$x = 6$

(b) $\frac{4x-3}{5} = 1$

$4x - 3 = 5$

$4x = 8$

$x = 2$

(c) $x^2 - 4x = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$

(d) $3x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(3)(5)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 - 60}}{6}$$

$$= \frac{2 \pm \sqrt{-56}}{6}$$