Number Properties

1. Natural Numbers = N = {1,2,3,...}
2. Whole Numbers = W = {0,1,2,3,...}
3. Integers = Z = {...,-3,-2,-1,0,1,2,3,...}
4. Rational Numbers = Q = $\left\{ \dfrac{p}{q} \mid p,q \in Z, q \neq 0 \right\}$
5. Irrational Numbers = I = {numbers that are not a ratio of two non-zero integers: $\sqrt{2}, \sqrt{5}, \pi$,...}
6. Real Numbers = R = Q union I = Q $\cup$ I

Let $a,b,c,d,x \in$ R and $i, j, k, n, m \in$ N

7. $a+b = b+a;\ ab = ba$
8. $a+0 = a = 0+a;\ a\bullet 1 = a = 1\bullet a$
9. $n\bullet a = (a+a+...+a);\ a^n = a\bullet a\bullet a...\bullet a$
10. $a(b+c) = ac+bc;\ (a+b)^2 = a^2 + 2ab + b^2;\ (a+b)(c+d) = ac+bd+bc+ad$
11. $-(a-b) = b-a$
12. $a = \dfrac{b}{c}$ if and only if $ac = b$
13. NEVER divide by 0.
14. $\dfrac{a}{1} = a$
15. $\dfrac{a}{a} = 1,\ a \neq 0$
16. $\dfrac{ab}{a} = b;\ a \neq 0$
17. $\dfrac{1}{\frac{a}{b}} = \dfrac{b}{a}$
18. $\dfrac{0}{a} = 0;\ a \neq 0$
19. $\dfrac{a}{b}\bullet\dfrac{c}{d} = \dfrac{ac}{bd};\ b \neq 0 \neq d$
20. $\dfrac{a}{b}+\dfrac{c}{d} = \dfrac{ad}{bd}+\dfrac{bc}{bd} = \dfrac{ad+bc}{bd};\ b \neq 0 \neq d$
21. $ab = 0$ if and only if $a = 0$ or $b = 0$.

22. If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

23. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots = \sum_{i=0}^{\infty} x^i$

DEF. C_m^n is the possible number of ways (combinations) to choose m elements from a set of n elements. (Order does not matter.)

24. $\displaystyle\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2} = C_2^{n+1}$

25. $C_m^n = \dfrac{n!}{m!(n-m)!}$; $\displaystyle\sum_{j=0}^{n} C_j^n = 2^n$; $C_m^n = C_{n-m}^n$

26. $\displaystyle\sum_{k=m}^{n} (ak+b) = \dfrac{(n-m+1)(am+b+an+b)}{2}$

27. $\displaystyle\sum_{i=0}^{n} r^i = \dfrac{r^{n+1}-1}{r-1}$, $r \neq 1$

DEF. $|a|$ is the distance a is from 0 .

28. $|a| = 0$ if and only if $a = 0$.

29. $|a| \geq 0$

30. $\sqrt{a^2} = |a|$