MATH 108 Sochacki Exam 1
Sept. 30, 2013 Take Home Part 50 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit. Your work must represent the question asked. You may NOT use computers. You may use your notes or text. Your work must be neat or I will not grade your test.

I have neither received nor given help on this exam. Mon Key
(Signature) (1 point)

1. Show whether or not there is a pattern for the numbers $1, \varphi, 2\varphi^2, 3\varphi^3, 4\varphi^4, 5\varphi^5, \ldots$ (6 points)

1

φ

$2\varphi^2 = 2(\varphi+1) = 2\varphi+1$

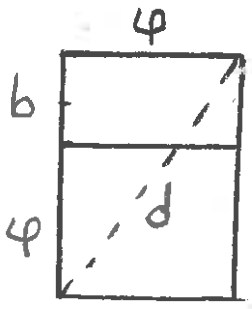
$3\varphi^3 = 3(\varphi^2+\varphi) = 3(\varphi+1+\varphi) = 3(2\varphi+1) = 6\varphi+3$

$4\varphi^4 = 4(\varphi^3+\varphi^2) = 4(2\varphi+1+\varphi+1) = 4(3\varphi+2) = 12\varphi+8$

$5\varphi^5 = 5(\varphi^4+\varphi^3) = 5(5\varphi+3) = 25\varphi+15$

$6\varphi^6 = 6(\varphi^5+\varphi^4) = 6(8\varphi+5) = 48\varphi+30$

2. Give the length of the diagonal in the golden rectangle below in terms of φ with no exponents on φ. (6 points)



$\frac{\varphi}{b} = \varphi$

$\varphi = b\varphi$

$b = 1$

$d^2 = (1+\varphi)^2 + \varphi^2$

$= 1+2\varphi+\varphi^2+\varphi^2$

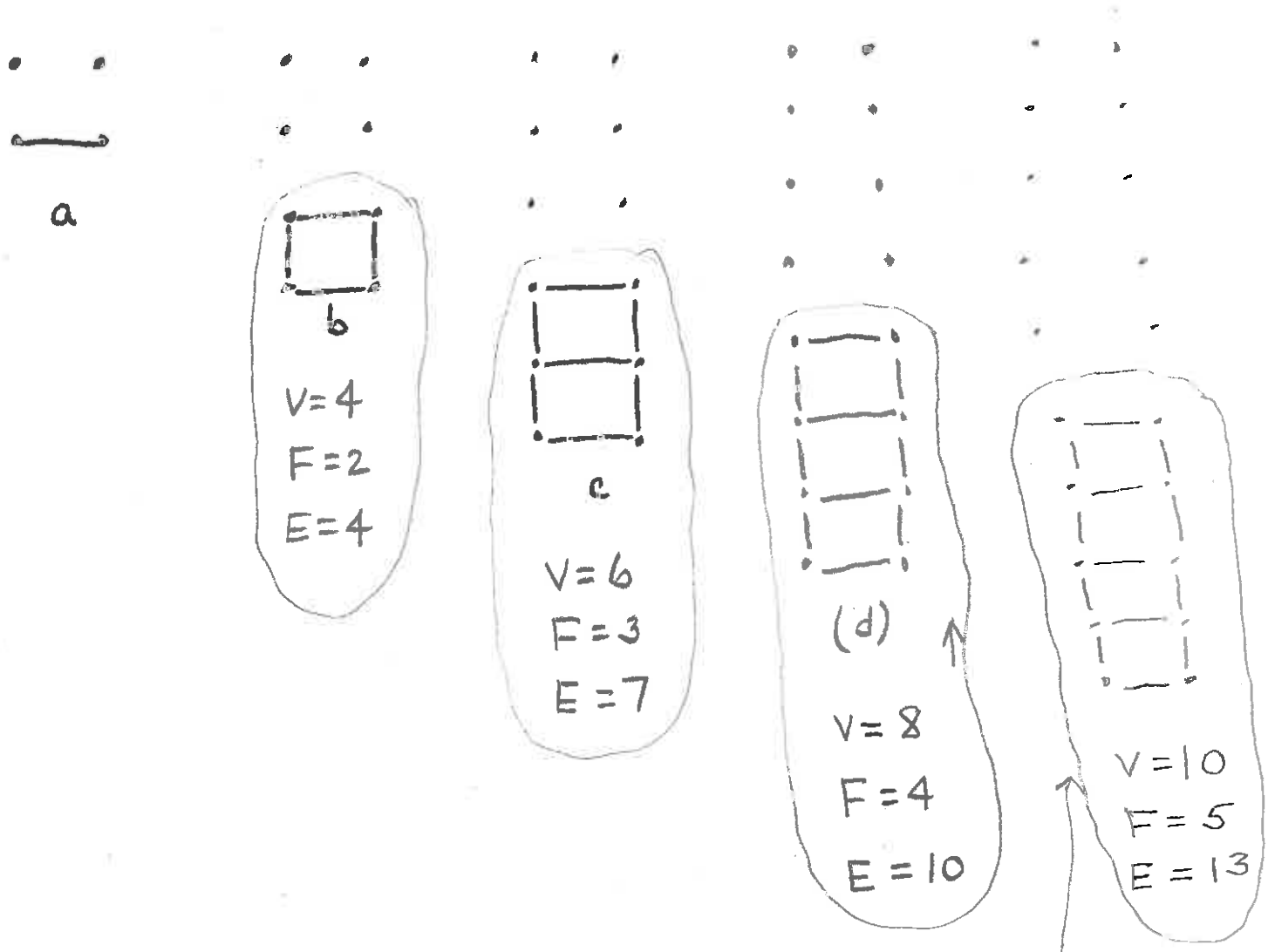
$= 1+2\varphi+2\varphi^2$

$= 1+2\varphi+2(\varphi+1)$

$= 4\varphi+3$

$d = \sqrt{4\varphi+3}$

3. Continue the pattern below shown in the first 3 figures for two more figures and then give a formula for the number of vertices, edges and faces from one pattern to the next for all figures in the pattern. (12 points)

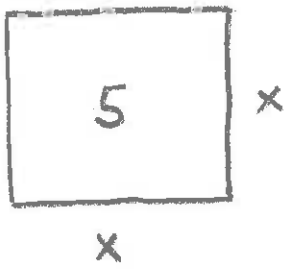


$V = 2n$

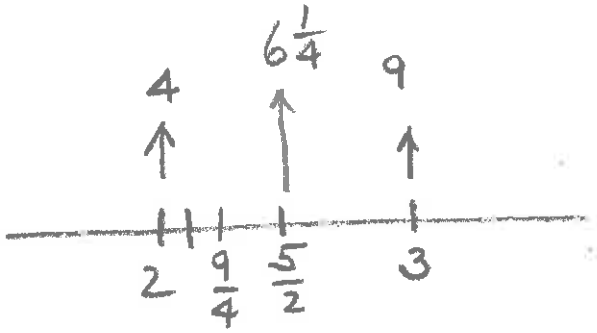
$F = n$

$E = V + F - 2 = 2n + n - 2 = 3n - 2$

4. The area of a square is 5. Develop two methods that approximate the length of the side of the square. Give at least four approximations to the length for both of your methods. (12 points)



$x^2 = 5$



$x_1 = \frac{2+3}{2} = \frac{5}{2}$ $\quad x_1^2 = \frac{25}{4} = 6\frac{1}{4}$

$x_2 = \frac{2+\frac{5}{2}}{2} = \frac{9}{4}$ $\quad x_2^2 = \frac{81}{16} = 5\frac{1}{16}$

$x_3 = \frac{2+\frac{9}{4}}{2} = \frac{17}{8}$ $\quad x_3^2 = \frac{289}{64} = 4\frac{33}{64}$

$x_4 = \frac{\frac{9}{4}+\frac{17}{8}}{2} = \frac{35}{16} = 2\frac{3}{16}$

$x^2 = 5$

$x^2 + x = 5 + x$

$x(x+1) = 5 + x$

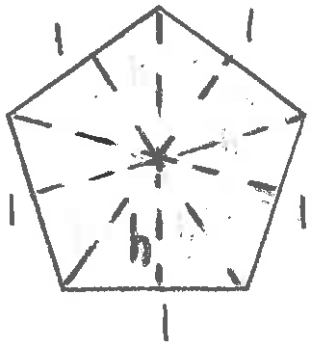
$x = \frac{5+x}{x+1}$

$x_1 = 2$ $\quad x_2 = \frac{7}{3}$

$x_3 = \frac{5+\frac{7}{3}}{\frac{7}{3}+1} = \frac{22}{10} = \frac{11}{5}$

$x_4 = \frac{5+\frac{11}{5}}{\frac{11}{5}+1} = \frac{36}{16} = \frac{9}{4}$

5. Give the area of the following figure. (6 points)

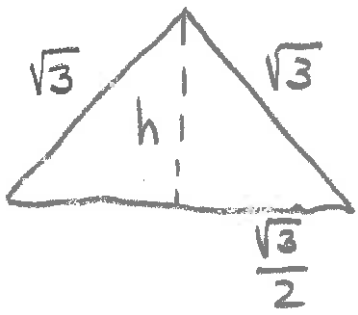


$$\text{Area} = 10\left(\frac{1}{2}h\right) = 5h$$

$$h \approx \frac{3}{4}$$

$$\text{Area} \approx 5\,\frac{3}{4} = \frac{15}{4}$$

6. Give the area of an equilateral triangle whose side length is $\sqrt{3}$. (6 points)



$$h^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \left(\sqrt{3}\right)^2$$

$$h^2 + \frac{3}{4} = 3$$

$$h^2 = 3 - \frac{3}{4} = \frac{9}{4}$$

$$h = \frac{3}{2}$$

$$\text{area} = \frac{1}{2}\sqrt{3}\,\frac{3}{2}$$

$$= \frac{3\sqrt{3}}{4}$$