MATH 237 Bonus Points
Sept. 26, 2016

Name Mon Key
(Print)

All necessary work must be shown. Your work must represent the question asked. Your work must be neat or I will not grade your work. You may discuss this assignment with others, but all work turned in must be your own work. Turn in this sheet with your work.

1. Give the tangent line in parametric form and the concavity to the parametric curve $(t\sin t, 2t\cos t)$ at $t=\frac{\pi}{4}$.

2. Give the equation of the plane containing the point (1,1,3) that is normal to the line $\frac{x-2}{4}=2y+1=\frac{z+3}{5}$.

3. Give the area of the triangle containing the points (2,0,-1),(0,2,-3) and (1,-2,0) and the volume of the parallelepiped having the origin and these three points as adjacent vertices (corners).

1. $x = t\sin t,\ y = 2t\cos t \qquad x\left(\frac{\pi}{4}\right)=\frac{\pi\sqrt{2}}{8} \quad y\left(\frac{\pi}{4}\right)=\frac{\pi\sqrt{2}}{4}$

$x' = \sin t + t\cos t,\ y' = 2\cos t - 2t\sin t \qquad x'\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\pi\sqrt{2}}{8},\ y'\left(\frac{\pi}{4}\right)=\sqrt{2}-\frac{\pi\sqrt{2}}{4}$

$x'' = 2\cos t - t\sin t,\ y'' = -4\sin t - 2t\cos t \qquad x''\left(\frac{\pi}{4}\right)=\sqrt{2}-\frac{\pi\sqrt{2}}{8},\ y''\left(\frac{\pi}{4}\right)=-2\sqrt{2}-\frac{\pi\sqrt{2}}{4}$

$$\ell(t) = \left(\frac{\pi\sqrt{2}}{8}, \frac{\pi\sqrt{2}}{4}\right) + t\left(\frac{\sqrt{2}}{2}+\frac{\pi\sqrt{2}}{8},\ \sqrt{2}-\frac{\pi\sqrt{2}}{4}\right)$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} = \frac{\left(\sqrt{2}-\frac{\pi\sqrt{2}}{8}\right)\left(\sqrt{2}-\frac{\pi\sqrt{2}}{4}\right)-\left(-2\sqrt{2}-\frac{\pi\sqrt{2}}{4}\right)\left(\frac{\sqrt{2}}{2}+\frac{\pi\sqrt{2}}{8}\right)}{\left(\frac{\sqrt{2}}{2}+\frac{\pi\sqrt{2}}{8}\right)^3}$$

2. $P = (1,1,3) \qquad \frac{x-2}{4} = 2y+1 = \frac{z+3}{5} = t$

$x = 4t+2,\ y = \frac{t-1}{2},\ z = 5t-3$

$\bar{n} = \langle 4, \frac{1}{2}, 5\rangle$

$\bar{n}\cdot\langle x-1, y-1, z-3\rangle = \langle 4,\frac{1}{2},5\rangle\cdot\langle x-1,y-1,z-3\rangle = 0$

$4(x-1)+\frac{1}{2}(y-1)+5(z-3)=0$

$4x-4+\frac{1}{2}y-\frac{1}{2}+5z-15=0$

$4x+\frac{1}{2}y+5z = 4+\frac{1}{2}+15$

$8x+y+10z=39$

3. $P_1 = (2,0,-1)$, $P_2 = (0,2,-3)$, $P_3 = (1,-2,0)$

$\bar{u} = \overrightarrow{P_1P_2} = \langle -2, 2, -2 \rangle$

$\bar{v} = \overrightarrow{P_1P_3} = \langle -1, -2, 1 \rangle$

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -2 & 2 & -2 \\ -1 & -2 & 1 \end{vmatrix} = \bar{i}(-2) - \bar{j}(-4) + \bar{k}(6)$$

$$\|\bar{u} \times \bar{v}\| = \sqrt{(-2)^2 + (4)^2 + (6)^2} = \sqrt{56} \qquad A_T = \frac{1}{2}\sqrt{56} = \sqrt{14}$$

$\bar{u} = \langle 2, 0, -1 \rangle$ $\bar{v} = \langle 0, 2, -3 \rangle$, $\bar{w} = \langle 1, -2, 0 \rangle$

$$\text{Volume} = |\bar{w} \cdot (\bar{u} \times \bar{v})|$$

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 0 & -1 \\ 0 & 2 & -3 \end{vmatrix} = \bar{i}(2) - \bar{j}(-6) + \bar{k}(4)$$

$$= 2\bar{i} + 6\bar{j} + 4\bar{k}$$

$$\bar{w} \cdot (\bar{u} \times \bar{v}) = 2 - 12 = -10$$

$$\text{Volume} = 10$$