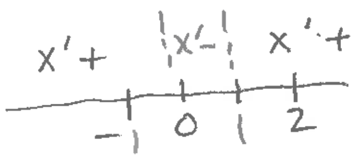
1. $x = 2t^3 - 3t^2$
$= t^2(2t-3)$

$x' = 6t^2 - 6t$
$= 6t(t-1)$

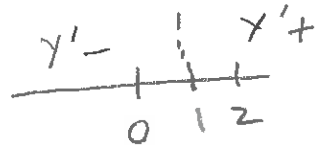
$x' = 0 \quad t = 0, 1$



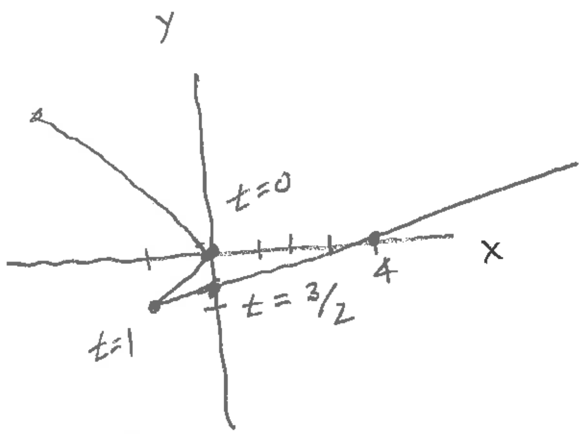
$y = t^2 - 2t$
$= t(t-2)$

$y' = 2t - 2$
$= 2(t-1)$

$y' = 0 \quad t = 1$



t	x	y
$-\infty$	$-\infty$	∞
0	0	0
1	-1	-1
$\frac{3}{2}$	0	$-\frac{3}{4}$
2	4	0
∞	∞	∞



$\bar{r}'(0) = 0\bar{i} - 2\bar{j}$

$\bar{r}'(1) = 0\bar{i} + 0\bar{j}$

$\bar{r}'(t) = (6t^2 - 6t)\bar{i} + (2t-2)\bar{j}$

$\|\bar{r}'(t)\| = \sqrt{(6t^2 - 6t)^2 + (2t-2)^2}$
$= \sqrt{[6t(t-1)]^2 + [2(t-1)]^2}$
$= |t-1|\sqrt{36t^2 + 4}$
$= 2|t-1|\sqrt{9t^2+1}$

$$\bar{T}(t) = \frac{6t(t-1)}{2|t-1|\sqrt{9t^2+1}}\bar{i} + \frac{2(t-1)}{2|t-1|\sqrt{9t^2+1}}\bar{j}$$

$$= \pm\left(\frac{3t}{\sqrt{9t^2+1}}\bar{i} \pm \frac{1}{\sqrt{9t^2+1}}\bar{j}\right)$$

$$= \pm\left(3t(9t^2+1)^{-1/2}\bar{i} + (9t^2+1)^{-1/2}\bar{j}\right)$$

$$\bar{T}'(t) = \pm\left(\left(3(9t^2+1)^{-1/2} - 27t^2(9t^2+1)^{-3/2}\right)\bar{i} - 9t(9t^2+1)^{-1/2}\bar{j}\right)$$

UGH! But we know $\bar{N} \perp \bar{T}$ so

$\bar{T} = \langle a, b \rangle$ and $\bar{N} = \pm \langle -b, a \rangle$

$$\bar{T}(t) = \pm \left(\frac{3t}{\sqrt{9t^2+1}}\, \bar{I} + \frac{1}{\sqrt{9t^2+1}}\, \bar{J} \right)$$

$$\bar{N}(t) = \pm \left(\frac{1}{\sqrt{9t^2+1}}\, \bar{J} - \frac{3t}{\sqrt{9t^2+1}}\, \bar{J} \right)$$

$$\bar{B}(t) = \bar{T}(t) \times \bar{N}(t) = \pm \bar{K}$$

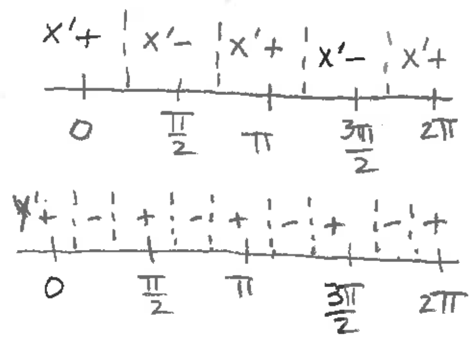
2. $\bar{r}(t) = \sin 2t\, \bar{i} + \sin 4t\, \bar{j}$

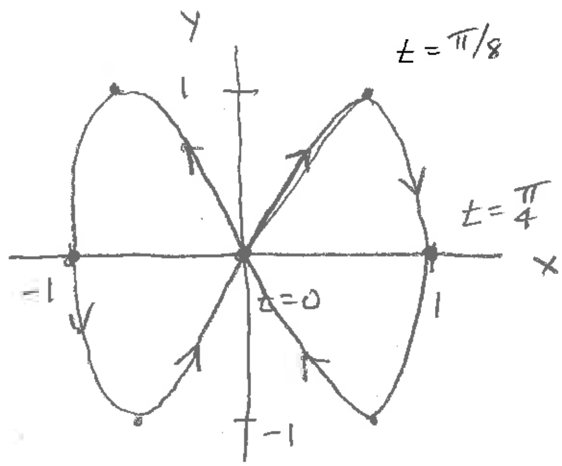
$$\bar{r}'(t) = 2\cos 2t\, \bar{i} + 4\cos 4t\, \bar{j} \Rightarrow \frac{ds}{dt} = \sqrt{4\cos^2 2t + 16\cos^2 4t}$$

$$\bar{r}''(t) = -4\sin 2t - 16\sin 4t\, \bar{j}$$

$x' = 2\cos 2t = 0 \quad t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$

$y' = 4\cos 4t = 0 \quad t = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots$





t	x	y
0	0	0
$\frac{\pi}{8}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{4}$	1	0
$\frac{3\pi}{8}$	$\frac{\sqrt{2}}{2}$	-1
$\frac{\pi}{2}$	0	0
$\frac{5\pi}{8}$	$-\frac{\sqrt{2}}{2}$	1
$\frac{3\pi}{4}$	-1	0
π	0	0

t	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$
$\frac{ds}{dt}$	2	4	2	2	4

$$\bar{T}(t) = \frac{2\cos 2t}{\sqrt{4\cos^2 2t + 16\cos^2 4t}}\, \bar{i} + \frac{4\cos 4t}{\sqrt{4\cos^2 2t + 16\cos^2 4t}}\, \bar{j}$$

$$\bar{N}(t) = \pm\left(\frac{4\cos 4t}{\sqrt{4\cos^2 2t + 16\cos^2 4t}}\, \bar{i} - \frac{2\cos 2t}{\sqrt{4\cos^2 2t + 16\cos^2 4t}}\, \bar{j} \right)$$

$$\bar{B}(t) = \pm \bar{k}$$

3. $\bar{r}'(t) = \langle t\cos t \, \bar{\imath} + t\sin t \, \bar{\jmath} + \sqrt{1-t^2} \, \bar{k}$

$$\frac{ds}{dt} = \|\bar{r}'(t)\| = \sqrt{t^2 + 1 - t^2} = 1$$

$$\bar{T}(t) = \bar{r}'(t)$$

$$\bar{T}'(t) = \bar{r}''(t) = \left\langle \cos t - t\sin t, \sin t + t\cos t, \frac{-t}{\sqrt{1-t^2}} \right\rangle = \bar{a}$$

$$\|\bar{T}'(t)\| = \|\bar{a}(t)\| = \sqrt{1 + t^2 + \frac{t^2}{1-t^2}}$$

$$= \sqrt{\frac{1-t^2 + t^2(1-t^2)}{1-t^2}} = \sqrt{\frac{1-t^4}{1-t^2}}$$

$$= \sqrt{1+t^2}$$

$$\bar{N}(t) = \frac{\left\langle \cos t - t\sin t, \sin t + t\cos t, \frac{-t}{\sqrt{1-t^2}} \right\rangle}{\sqrt{1+t^2}}$$

$$\bar{B}(t) = \frac{1}{\sqrt{1+t^2}} \begin{vmatrix} \bar{\imath} & \bar{\jmath} & K \\ t\cos t & t\sin t & \sqrt{1-t^2} \\ \cos t - t\sin t & \sin t + t\cos t & \frac{-t}{\sqrt{1-t^2}} \end{vmatrix}$$

$$\bar{r}(t) = \int \bar{r}'(t)\,dt = \left\langle \int \underbrace{t}_{u} \underbrace{\cos t\,dt}_{dv}, \int \underbrace{t}_{u} \underbrace{\sin t\,dt}_{dv}, \int \sqrt{1-t^2}\,dt \right\rangle$$

$$= \left\langle t\sin t - \int \sin t\,dt, \ -t\cos t - \int -\cos t\,dt, \right.$$

$$= \left\langle t\sin t + \cos t + c_1, \ -t\cos t + \sin t + c_2, \ \frac{1}{2} t\sqrt{1-t^2} + \arcsin(t) + c_3 \right\rangle$$

$$\bar{r}(0) = \langle 1 + c_1, c_2, c_3 \rangle = (0,0,0) \Rightarrow c_1 = -1, \ c_2 = 0, \ c_3 = 0$$

$$\bar{r}(t) = (t\sin t + \cos t - 1)\bar{i}\ (-t\cos t + \sin t)\bar{j} + \left(\frac{1}{2}t\sqrt{1-t^2} + \arcsin t\right)\bar{k}$$

$$\bar{r}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\sin\frac{1}{2} + \cos\frac{1}{2} - 1,\ \frac{-1}{2}\cos\frac{1}{2} + \sin\frac{1}{2},\ \frac{\sqrt{3}}{8} + \frac{\pi}{6}\right)$$

$$\bar{r}'\left(\frac{1}{2}\right) = \frac{1}{2}\cos\frac{1}{2}\,\bar{i} + \frac{1}{2}\sin\frac{1}{2}\,\bar{j} + \frac{\sqrt{3}}{2}\bar{k}$$

tangent line: $\ell(t) = \bar{r}\left(\frac{1}{2}\right) + t\,\bar{r}'\left(\frac{1}{2}\right)$

normal plane: $\left\langle x - \left(\frac{1}{2}\sin\frac{1}{2} + \cos\frac{1}{2} - 1\right),\ y - \left(\frac{-1}{2}\cos\frac{1}{2} + \sin\frac{1}{2}\right),\ z - \left(\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right)\right\rangle \cdot \left\langle \frac{1}{2}\cos\frac{1}{2},\ \frac{1}{2}\sin\frac{1}{2},\ \frac{\sqrt{3}}{2}\right\rangle = 0$