BP4 Monday Oct. 24, 2016

Let $z = f(x,y) = 4x^2 + 9y^2$ and $\bar{r}(t) = 3\sin 4t\,\bar{i} + 2\cos 4t\,\bar{j}$.

1. Show that $\bar{r}$ lies on a level curve of z.
2. Give the equation of the tangent plane to the graph of z at (0,0) and (1,2).
3. Suppose an ant is tracing out $\bar{r}$ in time t.
   (a) What is the ant's speed as it traces out this curve.
   (b) Show that the ant is always moving in a direction orthogonal to ∇z.
   (c) If z gives the temperature at any point (x,y), what is the change in the ant's temperature at $t = \frac{\pi}{8}$?
   (d) What direction should the ant walk at $t = \frac{\pi}{4}$ if it wants to decrease its temperature as fast as possible?

1. $f(\bar{r}(t)) = f(3\sin 4t, 2\cos 4t)$

$= 4(3\sin 4t)^2 + 9(2\cos 4t)^2 = 36\sin^2 4t + 36\cos^2 4t = 36$

2. $g(x,y,z) = 4x^2 + 9y^2 - z = 0$ level surface

$\nabla g = \langle 8x, 18y, -1\rangle$

$(0,0) \Rightarrow z = 0 \Rightarrow P = (0,0,0)$ $\bar{n} = \nabla g(0,0) = \langle 0,0,-1\rangle$

$\langle 0,0,-1\rangle \cdot \langle x-0, y-0, z-0\rangle = -z = 0$

$z = 0$

$(1,2) \Rightarrow z = 40 \Rightarrow P = (1,2,40)$ $\bar{n} = \nabla g(1,2) = \langle 8, 36, -1\rangle$

$\langle 8,36,-1\rangle \cdot \langle x-1, y-2, z-40\rangle = 0$

$8x - 8 + 36y - 72 - z + 40 = 0$

$8x + 36y - z = 40$

3. (a) $\bar{r}'(t) = 12\cos 4t\,\bar{i} - 8\sin 4t\,\bar{j}$

$\frac{ds}{dt} = \|\bar{r}'(t)\| = \sqrt{144\cos^2 4t + 64\sin^2 4t}$

(b) $\nabla z = \langle 8x, 18y\rangle = \langle 24\sin 4t, 36\cos 4t\rangle$

$\nabla z \cdot \bar{r}'(t) = \langle 24\sin 4t, 36\cos 4t\rangle \cdot \langle 12\cos 4t, -8\sin 4t\rangle$

$= 12 \cdot 24\sin 4t\cos 4t - 8 \cdot 36\cos 4t\sin 4t$

$= 0$

(c) $\frac{dz}{dt} = \nabla z \cdot \vec{r}\,'(t)$

$= \langle 8x, 18y \rangle \cdot \langle 12\cos 4t, -8\sin 4t \rangle$

$= \langle 24\sin 4t, 36\cos 4t \rangle \cdot \langle 12\cos 4t, -8\sin 4t \rangle$

$t = \frac{\pi}{8}$ $= \langle 24, 0 \rangle \cdot \langle 0, -8 \rangle = 0$

(d) $\nabla z = \langle 8x, 18y \rangle = \langle 24\sin 4t, 36\cos 4t \rangle$

$t = \frac{\pi}{4} \Rightarrow \langle 24, 0 \rangle$ ant should walk in x-direction.