BP 5 – Wed. Nov. 2

1. Give the tangent plane and normal line to the plane for $z=\sqrt{2y-x^2}$ at (1,2).
2. Give the tangent plane to $z=xy+y^2$ at (0,0) and (2,3). Explain the differences of these two planes in terms of z .
3. Give the tangent plane to $w=\ln(9-x^2-y^2-z^2)$ at (0,0,0) and (-1,2,3) Explain the differences of these two planes in terms of the w.

1. $z=(2y-x^2)^{\frac{1}{2}}$ $\quad z_x=\frac{1}{2}(2y-x^2)^{-\frac{1}{2}}(-2x)$ $\quad z_y=\frac{1}{2}(2y-x^2)^{-\frac{1}{2}}2$

$\bar{n}=\langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -1\rangle$ $\quad P=(1,2,\sqrt{3})$

Tangent Plane: $\langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -1\rangle \cdot \langle x-1, y-2, z-\sqrt{3}\rangle = 0$

$$-\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}y-z=\frac{-1}{\sqrt{3}}+\frac{2}{\sqrt{3}}-\sqrt{3}$$

$$x-y+\sqrt{3}z=\sqrt{3}-3$$

Normal Line: $(1,2,\sqrt{3})+t\langle\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -1\rangle \Rightarrow \langle 1-\frac{1}{\sqrt{3}}t, 2+\frac{1}{\sqrt{3}}t, \sqrt{3}-t\rangle$

2. $z=xy+y^2$ $\quad z_x=y$ $\quad z_y=x+2y$

(0,0): $\bar{n}=\langle 0,0,-1\rangle$ $\quad P=(0,0,0) \Rightarrow 0\cdot(x-0)+0\cdot(y-0)-(z-0)=0$

$z=0$ Tangent Plane is horizontal

Graph is flat near (0,0).

(2,3): $\bar{n}=\langle 3,8,-1\rangle$ $\quad P=(2,3,15) \Rightarrow 3(x-2)+8(x-3)-(z-15)=0$

$$3x+8y-z=15$$

3. $w=\ln(9-x^2-y^2-z^2)=\ln(9-(x^2+y^2+z^2))$

(0,0,0): $w=\ln 9$ $\quad w\leq \ln 9$ $\quad w=\ln 9$ only at (0,0,0)

$w_x=\frac{-2x}{9-x^2-y^2-z^2}$, $w_y=\frac{-2y}{9-x^2-y^2-z^2}$, $w_z=\frac{-2z}{9-x^2-y^2-z^2}$

$\nabla w(0,0,0)=\langle 0,0,0\rangle$

(-1,2,3): $w=\ln(9-1-4-9)=\ln(-5)$ impossible

(-1,2,3) is not in D(w).