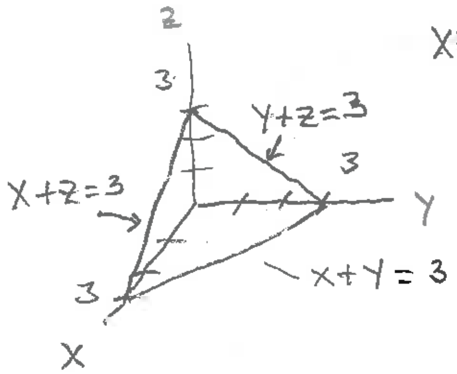
BP 7 – Due Mon. Nov. 28. Name Don Key Signature Key West

Do the following two problems on your own. You may use your text and notes.

1. Give the average value of $z = 12x$ over the region bounded by $x=0, y=0, z=0$ and $x+y+z=3$.

$$x+y+z=3 \Rightarrow z = 3-y-x \Rightarrow x = 3-y-z$$



$$\int_0^3 \int_0^{3-z} \int_0^{3-y-z} 12x \, dx\, dy\, dz$$

$$= \int_0^3 \int_0^{3-z} 6x^2 \Big|_0^{3-y-z} dy\, dz$$

$$= \int_0^3 \int_0^{3-z} 6(3-y-z)^2 \, dy\, dz$$

$$= \int_0^3 -2(3-y-z)^3 \Big|_0^{3-z} dz$$

$$= \int_0^3 (0 + 2(3-z)^3) \, dz$$

$$= -\frac{2}{4}(3-z)^4 \Big|_0^3 = \frac{1}{2} 3^4 = \frac{81}{2}$$

$$\text{AVG VAL} = \frac{\frac{81}{2}}{\frac{9}{2}} = 9$$

$$V = \int_0^3 \int_0^{3-x} \int_0^{3-y-x} dz\, dy\, dx$$

$$= \int_0^3 \int_0^{3-x} (3-y-x) \, dy\, dx$$

$$= \int_0^3 -\frac{(3-y-x)^2}{2} \Big|_0^{3-x} dx$$

$$= \int_0^3 \left(0 + \frac{(3-x)^2}{2}\right) dx$$

$$= -\frac{(3-x)^3}{6} \Big|_0^3 = \left(0 + \frac{3^3}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

2. Give the relative maximums and/or minimums of $z = x^4y^4 + 2x^2y^2 - 8$. Your work must show your answer.

$$\nabla z = \langle 4x^3y^4 + 4xy^2,\ 4x^4y^3 + 4x^2y \rangle$$

$$= \langle 4xy^2(x^2y^2+1),\ 4x^2y(x^2y^2+1) \rangle = \bar{0}$$

when $x=0, y=0$.
$y=a, x=b$

$$z = x^4y^4 + 2x^2y^2 - 8 \geq -8$$

So $z(0,a) = -8$ is a min,
$z(b,0) = -8$