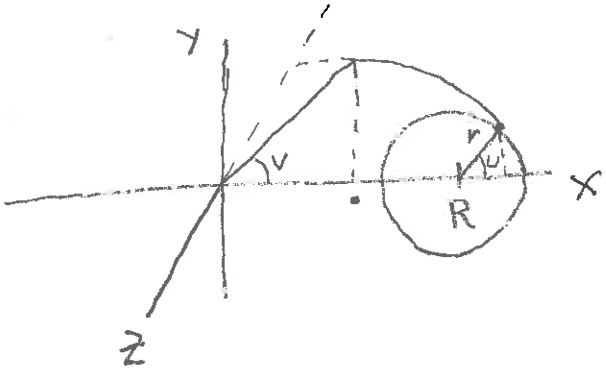

DONUT: $X = (R + r\cos u)\cos v \qquad Y = r\sin u$

$Z = (R + r\cos u)\sin v$

$$X^2 + Z^2 = (R + r\cos u)^2$$

$$\sqrt{X^2 + Z^2} = R + r\cos u$$

$$\left(\sqrt{X^2 + Z^2} - R\right) = r\cos u$$

$$\left(\sqrt{X^2 + Z^2} - R\right)^2 = r^2\cos^2 u = r^2 - r^2\sin^2 u = r^2 - Y^2$$

$$g(x, y, z) = Y^2 - r^2 + \left(\sqrt{X^2 + Z^2} - R\right)^2$$

$$\bar{r}(u, v) = X\bar{\imath} + Y\bar{\jmath} + Z\bar{k} = (R + r\cos u)\cos v\,\bar{\imath} + r\sin u\,\bar{\jmath} + (R + r\cos u)\sin v\,\bar{k}$$

$$\bar{r}_u = -r\sin u\cos v\,\bar{\imath} + r\cos u\,\bar{\jmath} - r\sin u\sin v\,\bar{k}$$

$$\bar{r}_v = -(R + r\cos u)\sin v\,\bar{\imath} + (R + r\cos u)\cos v\,\bar{k}$$

$$\bar{r}_u \times \bar{r}_v = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ -r\sin u\cos v & r\cos u & -r\sin u\sin v \\ -(R + r\cos u)\sin v & 0 & (R + r\cos u)\cos v \end{vmatrix}$$

$$= \bar{\imath}\,(r(R + r\cos u)\cos u\cos v + \bar{\jmath}\,(r(R + r\cos u)\sin u) + \bar{k}\,(r(R + r\cos u)\cos u\sin v)$$

$$\|\bar{r}_u \times \bar{r}_v\| = \sqrt{r^2(R + r\cos u)^2\cos^2 u + r^2(R + r\cos u)^2\sin^2 u}$$

$$= r(R + r\cos u)$$

$$SA = \int_0^{2\pi} \int_0^{2\pi} r\,(R + r\cos u)\,du\,dv$$

$$= r \int_0^{2\pi} \left(Ru + r\sin u \Big|_0^{2\pi} \right) dv$$

$$= r \int_0^{2\pi} 2\pi R\,dv \quad = \quad 2\pi r\ 2\pi R$$