MATH 237 Sochacki Exam 1 　　　　　　　　Name KEY WEST

Sep. 27, 2016 75 Points 　　　　　　　　(Print) (1 point)

All necessary work must be shown for credit. Part of your score is the work you show. Work must be NEAT. You may NOT use computers, notes or texts.

I have neither received nor given help on this exam. Don Key

(Signature) (2 points)

1. Give the UNIT normal to $2x-3y+4z=5$. (6 points)

$\bar{n} = \langle 2, -3, 4 \rangle$

$\bar{u} = \dfrac{\bar{n}}{\|\bar{n}\|}$

$\|\bar{n}\| = \sqrt{2^2+(-3)^2+4^2} = \sqrt{29} \Rightarrow \bar{u} = \left\langle \dfrac{2}{\sqrt{29}}, \dfrac{-3}{\sqrt{29}}, \dfrac{4}{\sqrt{29}} \right\rangle$

$= \dfrac{1}{\sqrt{29}}(2\bar{i} - 3\bar{j} + 4\bar{k})$

2. Give the angle between $\dfrac{x-3}{2} = \dfrac{2-y}{3} = 4z$ and $\bar{a} = 2\bar{j} - 8\bar{k}$. (6 points)

$\dfrac{x-3}{2} = \dfrac{2-y}{3} = 4z = t$

$x = 2t+3, \; -y = 3t-2, \; z = \frac{1}{4}t$

$y = -3t+2$

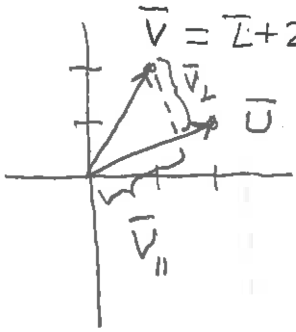
$\bar{v} = \langle 2, -3, \frac{1}{4} \rangle$

$\cos\theta = \dfrac{\bar{v}\cdot\bar{a}}{\|\bar{v}\|\,\|\bar{a}\|}$

$= \dfrac{-8}{\sqrt{2^2+(-3)^2+(\frac{1}{4})^2}\,\sqrt{2^2+(-8)^2}}$

$90^\circ < \theta < 180^\circ$

3. Let $\bar{u} = \langle 2, 1 \rangle$ and $\bar{v}$ be the vector determined by the point $P = (1, 2)$. Give the projection of $\bar{v}$ onto $\bar{u}$ ($\bar{v}_{\|}$) and the distance P is from the line given by $\bar{u}$. (6 points)



$\bar{v}_{\|} = \dfrac{\bar{u}\cdot\bar{v}}{\bar{u}\cdot\bar{u}}\,\bar{u} = \dfrac{4}{5}\langle 2, 1 \rangle$

$= \dfrac{8}{5}\bar{i} + \dfrac{4}{5}\bar{j}$

$\bar{v}_{\perp} = \bar{v} - \bar{v}_{\|} = \bar{i} + 2\bar{j} - \left(\dfrac{8}{5}\bar{i} + \dfrac{4}{5}\bar{j}\right)$

$= -\dfrac{3}{5}\bar{i} + \dfrac{6}{5}\bar{j}$

distance $= \|\bar{v}_{\perp}\| = \dfrac{1}{5}\sqrt{(-3)^2+6^2} = \dfrac{1}{5}\sqrt{45}$

4. Let $r = 4\sin\theta - 2$ be a polar curve. (18 points)

(a) Write the equivalent curve in rectangular coordinates.

$$r = 4\sin\theta - 2$$

$$r^2 = 4r\sin\theta - 2r$$

$$x^2 + y^2 = 4y - 2\sqrt{x^2+y^2}$$

(b) Give the equation of the tangent line to this polar curve at $\theta = \dfrac{\pi}{6}$.

$$x = r\cos\theta = 4\sin\theta\cos\theta - 2\cos\theta \qquad x\left(\tfrac{\pi}{6}\right) = \sqrt{3} - \sqrt{3} = 0$$

$$y = r\sin\theta = 4\sin^2\theta - 2\sin\theta \qquad y\left(\tfrac{\pi}{6}\right) = 1 - 1 = 0$$

$$\frac{dx}{d\theta} = 4\cos^2\theta - 4\sin^2\theta + 2\sin\theta \qquad \left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{6}} = 3 - 1 + 1 = 3$$

$$\frac{dy}{d\theta} = 8\sin\theta\cos\theta - 2\cos\theta \qquad \left.\frac{dy}{d\theta}\right|_{\theta=\frac{\pi}{6}} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

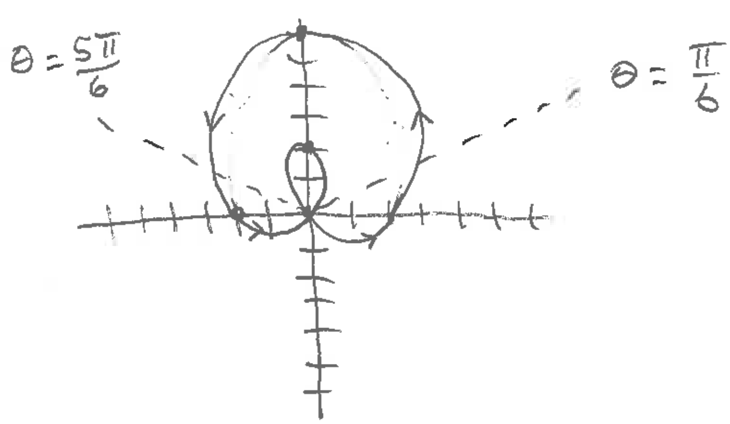
$$m = \frac{\sqrt{3}}{3} \quad \text{polar} \quad y = \frac{\sqrt{3}}{3}x$$

(c) Give the graph of this curve.

$$r = 0 \quad \sin\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

θ	r
0	-2
$\frac{\pi}{6}$	0
$\frac{\pi}{2}$	2
$\frac{5\pi}{6}$	0
π	-2

θ	r
$\frac{3\pi}{2}$	-6
2π	-2



5. Let $x = t^2 - 2t, y = t^3 - 27t$. (18 points)

(a) Give the equation of the tangent line in parametric form to this curve at $t = 2$.

$X(2) = 0 \quad Y(2) = 8 - 54 = -46$

$X' = 2t - 2, \quad Y' = 3t^2 - 27$

$X'(2) = 2 \quad Y'(2) = -15$

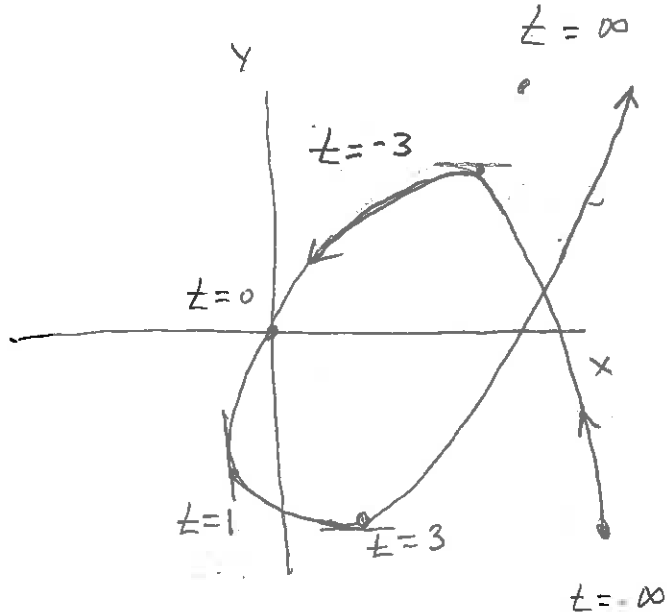
$\ell(t) = (0, -46) + t\langle 2, -15\rangle$

$X = 0 + 2t, \quad Y = -46 - 15t$

(b) Give the graph of this curve using limits and derivatives.

t	X	Y
$-\infty$	∞	$-\infty$
0	0	0
∞	∞	∞

t	X	Y
1	-1	-26
-3	15	54
3	3	54



$X' = 2t - 2 = 0 \quad t = 1$

$X'-$ | $X'+$

0 1 2

$Y' = 3t^2 - 27 = 0 \quad 3(t^2 - 9) = 9 \quad 3(t-3)(t+3) = 0 \quad t = \pm 3$

$Y'+$ | $Y'-$ | $Y'+$

-3 0 3

(c) Give the simplified arc length integral for this curve for $0 \le t \le 1$.

$X'(t)^2 = (2t-2)^2 = 4t^2 - 8t + 4$

$Y'(t)^2 = (3t^2 - 27)^2 = 9t^4 - 162t^2 + 27^2$

$X'(t)^2 + Y'(t)^2 = 9t^4 - 158t^2 - 8t + 733$

$S = \int_0^1 \sqrt{9t^4 - 158t^2 - 8t + 733}\, dt$

6. Give the volume of the parallelepiped determined by the vectors $\underset{\bar{U}}{<2,-1>}, \underset{\bar{V}}{<1,3>}$ and $\underset{\bar{W}}{2\bar{i}-\bar{j}+\bar{k}}$. (6 points)

$$<2,-1> \times <1,3> = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & 0 \\ 1 & 3 & 0 \end{vmatrix} = 7\bar{k}$$

$$|\bar{W} \cdot (\bar{U} \times \bar{V})|$$

$$2\bar{i} - \bar{j} + \bar{k} \cdot 7\bar{k} = 7$$

Volume $= 7$

7. Give the area of the triangle containing the points (1,-1,0),(0,2,-1) and (1,0,2). (6 points)

$P_1 = (1,-1,0) \quad P_2 = (0,2,-1) \quad P_3 = (1,0,2)$

$\bar{U} = \overrightarrow{P_1P_2} = <-1,3,-1> \quad \bar{V} = \overrightarrow{P_2P_3} = <1,-2,3>$

$$\bar{U} \times \bar{V} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 3 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \bar{i}(7) - \bar{j}(-2) + \bar{k}(-1)$$

$$= 7\bar{i} + 2\bar{j} - \bar{k}$$

$$A_T = \frac{1}{2}\|\bar{U} \times \bar{V}\| = \frac{1}{2}\sqrt{7^2+2^2+1^2}$$

$$= \frac{1}{2}\sqrt{54}$$

8. Give the point at which $\ell(t) = <2t+1, 4t, t-1>$ intersects $x+y+z=7$. (6 points)

$X = 2t+1, \quad Y = 4t, \quad Z = t-1$

$$X+Y+Z = (2t+1) + 4t + (t-1) = 7t = 7$$

$$t = 1$$

$X = 3, Y = 4, Z = 0$