MATH 237 Sochacki Exam 3
Nov. 15, 2016 In Class Part 75 Points

Name KEY WEST
(Print) (1 point)

All necessary work must be shown for credit. Part of your score is the work you show. Work must be NEAT. You may NOT use computers, notes or texts.

I have neither received nor given help on this exam. Don Key
(Signature) (2 points)

1. Give the value of $\int_{-1}^{0}\int_{-\pi}^{\frac{\pi}{2}} x\cos(xy)dydx$. (9 points)

$$\int_{-\pi}^{\frac{\pi}{2}} \underbrace{x\cos(\underbrace{xy}_{u})\,dy}_{du} = \sin(xy)\Big|_{-\pi}^{\frac{\pi}{2}} = \sin(\tfrac{\pi}{2}x) - \sin(-\pi x)$$

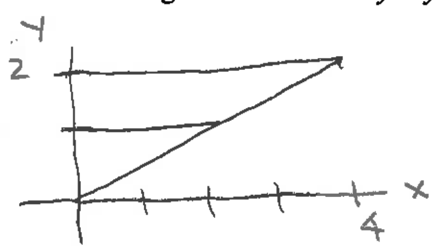
$$= \sin(\tfrac{\pi}{2}x) + \sin(\pi x)$$

$$\int_{-1}^{0}\int_{-\pi}^{\frac{\pi}{2}} x\cos(xy)\,dy\,dx = \int_{-1}^{0}\left(\sin(\tfrac{\pi}{2}x) + \sin(\pi x)\right)dx$$

$$= -\frac{2}{\pi}\cos(\tfrac{\pi}{2}x) - \frac{1}{\pi}\cos(\pi x)\Big|_{-1}^{0}$$

$$= \left(-\frac{2}{\pi} - \frac{1}{\pi}\right) - \left(\frac{1}{\pi}\right) = -\frac{4}{\pi}$$

2. Give the average value of $z = xy - y^2$ over $Q = \{(x,y)\,|\,0 \le x \le 2y, 0 \le y \le 2\}$. (9 points)




Area $= \frac{1}{2}bh = \frac{1}{2}\cdot 4\cdot 2 = 4$

$$\int_0^2\int_0^{2y} (xy - y^2)\,dx\,dy$$

$$= \int_0^2 \left(\frac{x^2}{2}y - xy^2\right)\Big|_0^{2y} dy$$

$$= \int_0^2 (2y^3 - 2y^3)\,dy = 0$$

Avg. Val. $= 0$

3. Let $f(x,y)=x^2+2xy+4y^2-4x+8y+20$. Give the equation of the tangent plane at the points where $grad(f)=\bar{0}$. (9 points)

$$\nabla f = \langle 2x+2y-4,\ 2x+8y+8 \rangle$$

$$2x+2y-4=0$$
$$2x+8y+8=0$$
$$-6y-12=0$$
$$-6y=12$$
$$y=-2$$
$$2x-4-4=0$$
$$2x=8$$
$$x=4$$

$$z=f(4,-2)=16-16+16-16-16+20=4$$

$$P=(4,-2,4)$$

$$\langle 0,0,-1\rangle \cdot \langle x-4, y+2, z-4\rangle = -z+4=0$$
$$z=4$$

$$\bar{n}=\langle 0,0,-1\rangle$$

4. Give the equation of the line normal to the level surface of $f(x,y,z)=\ln(x^2+y^2-z)$ at (3,4,5). What is the level surface? (9 points)

$$f(3,4,5)=\ln(9+16-5)=\ln(20)=\ln(x^2+y^2-z)$$
$$x^2+y^2-z=20$$
$$z=x^2+y^2-20$$

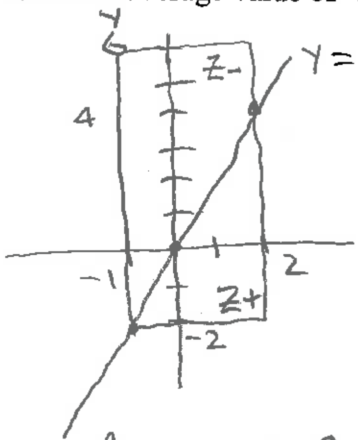
paraboloid

$$\nabla f = \left\langle \frac{2x}{x^2+y^2-z}, \frac{2y}{x^2+y^2-z}, \frac{-1}{x^2+y^2-z} \right\rangle$$

$$\nabla f(3,4,5) = \left\langle \frac{6}{20}, \frac{8}{20}, \frac{-1}{20} \right\rangle = \left\langle \frac{3}{10}, \frac{2}{5}, \frac{-1}{20} \right\rangle$$

$$\ell(t) = P + t\bar{v} = (3,4,5) + t\left\langle \frac{3}{10}, \frac{2}{5}, \frac{-1}{20} \right\rangle$$
$$= \left\langle 3+\frac{3}{10}t,\ 4+\frac{2}{5}t,\ 5-\frac{1}{20}t \right\rangle$$

5. Give the average value of $z = |2x - y|$ over $Q = \{(x,y) \mid -1 \le x \le 2, -2 \le y \le 6\}$. (9 points)




Area = 3·8 = 24

$$\int_{-1}^{2}\int_{-2}^{2x}(2x-y)\,dy\,dx - \int_{-1}^{2}\int_{2x}^{6}(2x-y)\,dy\,dx$$

$$= \int_{-1}^{2}\left(2xy-\frac{y^2}{2}\right)\Big|_{-2}^{2x}dx - \int_{-1}^{2}\left(2xy-\frac{y^2}{2}\right)\Big|_{2x}^{6}dx$$

$$= \int_{-1}^{2}\left[(4x^2-2x^2)-(-4x-2)\right]dx - \int_{-1}^{2}\left(\left(12x-18\right)-\left(4x^2-2x^2\right)\right)$$

$$= \int_{-1}^{2}(2x^2+4x+2)\,dx - \int_{-1}^{2}(12x-18-2x^2)\,dx$$

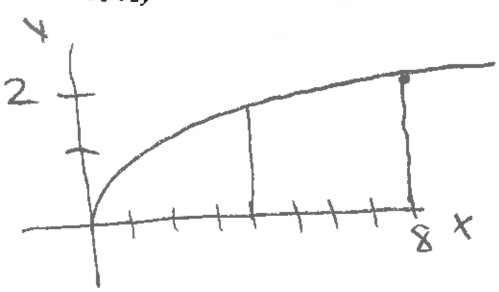
$$= \left(\frac{2}{3}x^3+2x^2+2x\right)\Big|_{-1}^{2} - \left(6x^2-18x-\frac{2}{3}x^3\right)\Big|_{-1}^{2}$$

$$= \left(\frac{16}{3}+8+4\right)-\left(\frac{-2}{3}+2-2\right) - \left(\left(24-36-\frac{16}{3}\right)-\left(6+18+\frac{2}{3}\right)\right)$$

$$= 18-(-12-6-24) = 60$$

$$\text{Avg. Val} = \frac{60}{24} = \frac{5}{2}$$

6. Evaluate $\int_0^2 \int_{2y^2}^8 y\sin(x^2)\,dx\,dy$. (9 points)




$$x = 2y^2 \Rightarrow y^2 = \frac{x}{2} \Rightarrow y = \sqrt{\frac{x}{2}}$$

$$\int_0^8\int_0^{\sqrt{\frac{x}{2}}} y\sin(x^2)\,dy\,dx$$

$$= \int_0^8 \frac{y^2}{2}\sin(x^2)\Big|_0^{\sqrt{\frac{x}{2}}}dx$$

$$= \int_0^8 \frac{x}{4}\sin(x^2)\,dx = \frac{1}{8}\int_0^8 2x\sin(x^2)\,dx$$

$$= \frac{-1}{8}\cos(x^2)\Big|_0^8 = \frac{-1}{8}\cos 64 + \frac{1}{8}$$

7. Give the volume of the solid bounded by $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$. (9 points)

$$z = 18 - x^2 - y^2 = x^2 + y^2$$

$$18 = 2x^2 + 2y^2$$

$$9 = x^2 + y^2$$

$$r^2 = 9, \quad r = 3$$

$$\int_0^{2\pi}\int_0^3\int_{r^2}^{18-r^2} dz\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^3 z\Big|_{r^2}^{18-r^2} r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^3 ((18-r^2)-r^2)\, r\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^3 (18-2r^2)\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 (18r - 2r^3)\,dr\,d\theta$$

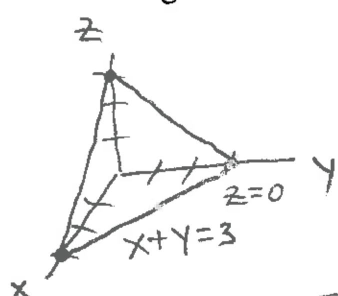
$$= \int_0^{2\pi} 9r^2 - \frac{1}{2}r^4\Big|_0^3 d\theta$$

$$= \int_0^{2\pi} \left(81 - \frac{81}{2}\right) d\theta$$

$$= 2\pi\left(\frac{81}{2}\right)$$

$$= 81\pi$$

8. Give the average value of $z = 12x$ over the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 3$. (9 points)




$$\frac{\int_0^3\int_0^{3-x}\int_0^{3-x-y} 12x\,dz\,dy\,dx}{\int_0^3\int_0^{3-x}\int_0^{3-x-y} dz\,dy\,dx} = \frac{\frac{81}{2}}{\frac{9}{2}} = \boxed{9}$$

$$27x^2 - 12x^3 + \frac{3}{2}x^4\Big|_0^3$$

$$\int_0^3\int_0^{3-x}\int_0^{3-x-y} dz\,dy\,dx$$

$$= \int_0^3\int_0^{3-x} z\Big|_0^{3-x-y} dy\,dx$$

$$= \int_0^3\int_0^{3-x} (3-x-y)\,dy\,dx$$

$$= \int_0^3 \left(3y - xy - \frac{y^2}{2}\Big|_0^{3-x}\right) dx$$

$$= \int_0^3 9 - 3x - 3x + x^2 - \frac{(3-x)^2}{2}\,dx = 9x - 3x^2 + \frac{x^3}{3} + \frac{(3-x)^3}{6}\Big|_0^3 = \left(9 - \frac{3^3}{6}\right) = 9 - \frac{9}{2} = \frac{9}{2}$$

$$= \int_0^3\int_0^{3-x} 12xz\Big|_0^{3-x-y} dy\,dx = \int_0^3\int_0^{3-x} (36x - 12x^2 - 12xy)\,dy\,dx$$

$$= \int_0^3 \left(36xy - 12x^2y - 6xy^2\Big|_0^{3-x}\right) dx$$

$$= \int_0^3 36x(3-x) - 12x^2(3-x) - 6x(3-x)^2\,dx$$

$$= \int_0^3 [6x(3-x)(6-2x-(3-x))]\,dx$$

$$= \int_0^3 6x(3-x)^2\,dx = \int_0^3 (54x - 36x^2 + 6x^3)\,dx$$