1(a) $f(x,y,z) = yxz^2 + \ln\sqrt{x^2+y^2} = xyz^2 + \frac{1}{2}\ln(x^2+y^2)$

$$f_x = yz^2 + \frac{x}{x^2+y^2}$$

$$f_y = xz^2 + \frac{y}{x^2+y^2}$$

$$f_z = 2xyz$$

$f_z = 0 \Rightarrow x = 0,\ y = 0$ or $z = 0$

$xf_x - yf_y = \dfrac{x^2 - y^2}{x^2+y^2} = 0 \Rightarrow y^2 - x^2 = 0 \Rightarrow y = \pm x$

If $z = 0$ then $y = 0$ and $x = 0$

If $y = 0$ and $z = c$ then $x = 0$

If $x = 0$ and $z = c$ then $y = 0$

If $x = a$ then $y = \pm a$ and $\pm az^2 + \dfrac{a}{2a^2} = 0$

$$\pm 2a^3z^2 + a = 0$$

$$z^2 = \pm\frac{a}{2a^3} = \pm\frac{1}{2a}$$

$$z = \pm\sqrt{\frac{1}{2a}}$$

2 (b) $z = 2x^2 + 8xy + 2y^2 - 6x + 8$

$z_x = 4x + 8y - 6$

$z_y = 8x + 4y$

$z_y = 0 \Rightarrow y = -2x$

$z_x = 0 \Rightarrow 4x + 8(-2x) - 6 = 0$

$-12x - 6 = 0$

$x = -\frac{1}{2} \Rightarrow y = -2(-\frac{1}{2}) = 1$

$z_{xx} = 4 \quad z_{yy} = 4 \quad z_{xy} = 8$

$z_{xx} z_{yy} - z_{xy}^2 = 4 \cdot 4 - 8^2 < 0$

saddle at $(-\frac{1}{2}, 1)$

(c) $f(x,y) = x^3 - 9xy + y^3$

$f_x = 3x^2 - 9y$

$f_y = -9x + 3y^2$

$f_x = 0 \Rightarrow y = \frac{1}{3}x^2$

$f_y = 0 \Rightarrow -9x + 3\left(\frac{1}{3}x^2\right)^2 = -9x + \frac{1}{3}x^4 = 0$

$27x - x^4 = 0$

$x(27 - x^3) = 0$

$x = 0,\ x = 3$

$y = 0,\ y = 3$

$f_{xx} = 6x,\ f_{yy} = 6y,\ f_{xy} = -9$

$D(0,0) = 0 \cdot 0 - (-9)^2 < 0$ saddle at $(0,0)$

$D(3,3) = 18 \cdot 18 - (-9)^2 > 0$ $f_{xx} > 0$ rel. min. at $(3,3)$

$f(3,3) = 27 - 81 + 27 = -27$

3. $z = e^{x^2+y^2}$ $\quad g(x,y) = xy - 1$

$$L(x,y,\lambda) = e^{x^2+y^2} - \lambda(xy-1)$$

$$\frac{\partial L}{\partial x} = 2xe^{x^2+y^2} - \lambda y = 0$$

$$\frac{\partial L}{\partial y} = 2ye^{x^2+y^2} - \lambda x = 0$$

$$\frac{\partial L}{\partial \lambda} = 1 - xy = 0$$

$$y\frac{\partial L}{\partial x} - x\frac{\partial L}{\partial y} = -\lambda y^2 + \lambda x^2 = -\lambda(y^2 - x^2) = 0$$

$$\lambda = 0, \quad y = \pm x$$

$$1 - xy = 1 \pm x^2 = 0 \Rightarrow x^2 = 1, \quad x = \pm 1$$

$$y = \pm 1$$

$x = 1, y = 1 \quad z = e^2$

$x = -1, y = -1 \quad z = e^2$

4. $f(x,y,z) = e^{x^2+y^2-z}$ $\quad g(x,y,z) = 2x+4y-6z-8$

$$L(x,y,z,\lambda) = e^{x^2+y^2-z} - \lambda(2x+4y-6z-8)$$

$$\begin{cases} \dfrac{\partial L}{\partial x} = 2xe^{x^2+y^2-z} - 2\lambda = 0 \\ \dfrac{\partial L}{\partial y} = 2ye^{x^2+y^2-z} - 4\lambda = 0 \end{cases}$$

$$\frac{\partial L}{\partial z} = -e^{x^2+y^2-z} + 6\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 8+6z-4y-2x = 0$$

$$y\frac{\partial L}{\partial x} - x\frac{\partial L}{\partial y} = -2\lambda y + 4\lambda x = 0$$

$$-2\lambda(y-2x) = 0$$

$$\lambda = 0,\ y = 2x$$

$$-e^{x^2+y^2-z} + 6\lambda = 0 \Rightarrow \lambda \neq 0$$

$$y = 2x \Rightarrow -e^{x^2+4x^2-z} + 6\lambda = 0$$

$$\lambda = \frac{1}{6}e^{5x^2-z}$$

also

$$\lambda = xe^{x^2+y^2-z} = xe^{5x^2-z}$$

$$\lambda = \frac{1}{2}ye^{x^2+y^2-z} = xe^{5x^2-z}$$

$$\lambda = \frac{1}{6}e^{5x^2-z} = xe^{5x^2-z} \Rightarrow x = \frac{1}{6},\ y = \frac{1}{3}$$

$$8+6z-\frac{4}{3}-\frac{1}{3} = 0 \Rightarrow 6z = \frac{5}{3}-8 \Rightarrow z = \frac{\frac{5}{3}-8}{6}$$

5. $P = (0,1,2)$ $\quad g(x,y,z) = x^2+y^2-z-4$

$$d(P,g) = \sqrt{(x-0)^2+(y-1)^2+(z-2)^2}$$

$$S(x,y,z) = x^2+(y-1)^2+(z-2)^2$$

$$L(x,y,z,\lambda) = x^2+(y-1)^2+(z-2)^2-\lambda(x^2+y^2-z-4)$$

$$\frac{\partial L}{\partial x} = 2x-2\lambda x = 0 \Rightarrow 2x(1-\lambda)=0 \Rightarrow x=0, \lambda=1$$

$$\frac{\partial L}{\partial y} = 2(y-1)-2\lambda y = 0 \Rightarrow 2y(1-\lambda)=2 \Rightarrow \lambda \neq 1 \quad y = \frac{1}{1-\lambda}$$

$$\frac{\partial L}{\partial z} = 2(z-2)+\lambda = 0 \Rightarrow 2z-4+\lambda=0 \Rightarrow z = \frac{4-\lambda}{2}$$

$$\frac{\partial L}{\partial \lambda} = 4+z-x^2-y^2=0 \Rightarrow 4+\frac{4-\lambda}{2}-0^2-\left(\frac{1}{1-\lambda}\right)^2=0$$

solve for λ.

6. Volume $= xyz$ $\qquad z+xy=32$

$$L(x,y,z) = xyz-\lambda(z+xy-32)$$

$$\frac{\partial L}{\partial x} = yz-\lambda y=0 \Rightarrow y(z-\lambda)=0 \Rightarrow z=\lambda$$

$$\frac{\partial L}{\partial y} = xz-\lambda x=0 \Rightarrow x(z-\lambda)=0 \Rightarrow z=\lambda$$

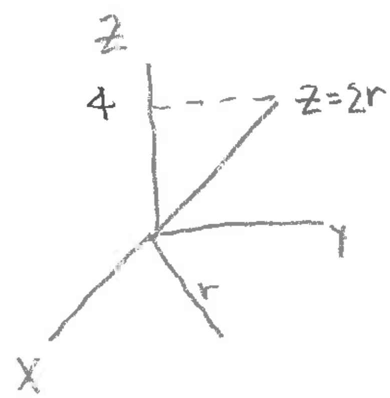
$$\frac{\partial L}{\partial z} = xy-\lambda=0 \Rightarrow xy=\lambda \Rightarrow xy=\lambda=z$$

$$\frac{\partial L}{\partial \lambda} = 32-xy-z=0 \Rightarrow 32-z-z=0 \Rightarrow z=16$$

$xy=16$

$V=16\cdot 16$

7. (a) $z = 2r \qquad 0 \le z \le 4$



$$\bar{r}(\theta, r) = r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j} + 2r\,\bar{k}$$

$$\frac{\partial}{\partial\theta}\bar{r} = \bar{r}_\theta = -r\sin\theta\,\bar{i} + r\cos\theta\,\bar{j}$$

$$\frac{\partial}{\partial r}\bar{r} = \bar{r}_r = \cos\theta\,\bar{i} + \sin\theta\,\bar{j} + 2\bar{k}$$

$$\bar{r}_\theta \times \bar{r}_r = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -r\sin\theta & r\cos\theta & 0 \\ \cos\theta & \sin\theta & 2 \end{vmatrix}$$

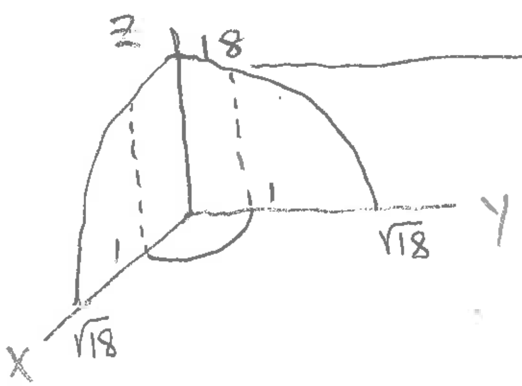
$$= \bar{i}\,(2r\cos\theta) - \bar{j}\,(-2r\sin\theta) + \bar{k}\,(-r)$$

$$= 2r\cos\theta\,\bar{i} + 2r\sin\theta\,\bar{j} - r\,\bar{k}$$

$$\|\bar{r}_\theta \times \bar{r}_r\| = \sqrt{4r^2 + r^2} = \sqrt{5r^2} = r\sqrt{5}$$

$$SA = \int_0^{2\pi}\int_0^{2} r\sqrt{5}\;dr\,d\theta = 2\pi\sqrt{5}\,\frac{r^2}{2}\Big|_0^2 = 4\pi\sqrt{5}$$

(c) $z = 18 - x^2 - y^2, \quad x^2 + y^2 = 1$



$$\rightarrow 18 - x^2 - y^2 = 18 - 1 = 17$$

$$z = 17$$

$$SA_{cylinder} = \pi r^2 h = \pi(1)^2\,17$$

$$= 17\pi$$

SA_{TOP}

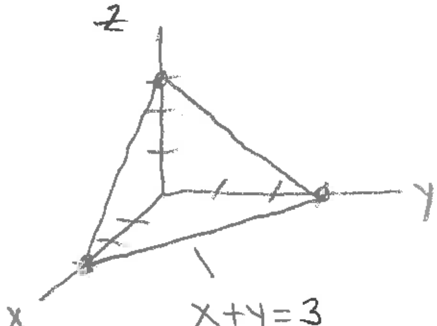
$$z = 18 - x^2 - y^2$$

$$z_x = -2x \,,\quad z_y = -2y$$

$$SA = \iint\limits_{\substack{\text{circle} \\ r=1}} \sqrt{(-2x)^2 + (2y)^2 + 1}\ dx\,dy$$

$$= \int_0^{2\pi} \int_0^1 \sqrt{4r^2+1}\ r\,dr\,d\theta$$

(e) $z = 3 - x - y$



$$SA = 3\left(\tfrac{1}{2}\cdot 3\cdot 3\right) + SA_{TOP}$$

SA_{TOP}

$$z_x = -1 \,,\quad z_y = -1$$

$$SA = \int_0^3 \int_0^{3-x} \sqrt{(-1)^2 + (-1)^2 + 1^2}\ dy\,dx$$

<u>OR</u> Find SA_{TOP} using cross products.